

## The Whole Thing is a (Now) Number

*(Foundations Paper 2)*

D.J. Huntington Moore

### Abstract

All of the traditional sciences of our day are founded on analytic arithmetic, a relatively recent development. The ancient Greeks practiced a synthetic kind of arithmetic. Instead of scalar magnitudes, they resorted to geometric line segments. The product of line segments could be a rectangular area, or a volume, for example. Theirs was a *geometric* arithmetic. In this paper, based on previous work, we report a new kind of science using non-ordered synthetic number. We call them *now numbers*. The number system is binary with the two elementary parts corresponding to the ancient ontological gender construct. It turns out that there are four elementary now numbers corresponding to the binary genders fm, ff, fm, and mm, reminiscent of the implicit gendering of the four classical elements.

We show how these four now numbers can be interpreted as providing the geometric semantics of the four letters of the genetic code. We can also show how now numbers exhibit spacetime like and quantum mechanics like semantics. The synthetic role of now numbers is to construct organisms capable of attaining and maintaining a coherent individual Nowness. Thus, as the Pythagoreans declared, perhaps you and even the whole thing are composite now numbers.

### Keywords:

Quantum Algebra, geometric algebra, dispositions, genetic code, *Analysis Situs*, Stoicism, Leibniz, hyper-complex numbers

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## 1 Introduction

The first paper in this series (D. J. Moore, *Now Machines* 2013a), characterised the traditional science and mathematics as *diachronic*, specialising in studying relations between the *a priori* and the *a posteriori*. The empirical sciences endeavour to predict the future outcomes from accumulated past experience, axiomatic mathematic deduces new theorems from an already postulated axiomatic base. The paper proposed another kind of science that would study the Parmenidean world in between the before and the after, in between the antecedent and its logical and phenomenal consequents. Such a science can be characterised as *synchronic*.

There are two possible kinds of science, the categorical and the dispositional. Present day sciences, including axiomatic mathematics, are categorical in nature. Such sciences are forcibly based on abstraction and explain reality in analytic, reductionist, and dualist terms. A characterising feature of all modern science is rampant specialisation where, true to the dictates of the categorical paradigm, each specialised field of knowledge explodes into ever increasing finer specialisation. The author likes to refer to these traditional sciences, including axiomatic mathematics, as *left side* sciences. This paper is one of a series and presents the “right side” alternative. The paper outlines the epistemological foundations of a non-categorical right side science and mathematics. Abstraction is the biggest causality in this venture and perhaps the hardest to comprehend for us moderns. All of our traditional sciences as well as Analytic Philosophy are founded on abstraction. Hegel once mischievously claimed that abstract thinking was the affair of the uneducated, uncultivated masses (Hegel 1808). This paper presents the alternative to abstract thinking—how to think universally and, above all, generically.

The generic alternative to abstraction is to change the nature of the scientific object of study. To be avoided at all cost as objects of explicit study are the accidental entities that may or may not populate our world. Also to be avoided is to construct abstract taxonomic, categorical structures in order to explain the accidental. Such endeavours are the affair of the traditional analytic and empirical sciences. Instead, the entity under consideration becomes not the entity but the necessary apparatus needed to comprehend it. Even more importantly is the apparatus needed for this world of the accidental and the phenomenal to behave in a coherent and rational manner. There are two kinds of apparatus are at play here, one individual and one impersonal. The mechanism by which local rationality and global rationality interact, mix, and even adhere, must be a central theme of non-categorical science. The theme is central to this series of papers.

One approach to developing a right side science is to see it as providing a “Theory of Signs” in the form of what Charles Sander Peirce called *semiotics*. In fact, Peirce’s writings

provided the initial inspiration for this project and have provided a constant point of reference. However, despite Peirce's deep insights, the author has been obliged to abandon his key doctrinal theme, his dogmatic insistence on the primacy of triadic structures *per se*. The triads are there, triads are everywhere, but they are only part of a more universal structure. Peirce remarked that most philosophical doctrines and even theological doctrines are based on triads. In the search for a replenished vocabulary, we briefly turn to Karl Jung. For Jung the doctrines were all *Three-plus-One structures*. The Peirce and Jung perspectives are like the Protestant view of the Trinity versus the Orthodox and Catholic. For the Protestant there is no flesh or blood in the Trinity. One could say that there is only the *Three-plus-None*. This is all just a picturesque and dramatic way of saying that we will be presenting a Three-plus-One doctrine as distinct from Peirce's Three-plus-None version.

Peirce made substantial contributions to symbolic logic. He is responsible for the universal and existential quantifiers. He is also known for his Existential Graphs. In unpublished work in 1880, he showed that the propositional calculus could be expressed with one single primitive, the NOR operator sometimes called the *Peirce dagger*<sup>1</sup>. All of these contributions were to what we would call left side logic.

As we know, Peirce was also interested in another kind of logic, a *semiotic logic*, a logic of signs and their meanings. Now for the author, this moving from symbolic logic to semiotic logic is much more dramatic than crossing the Rubicon.

In the previous paper, the author borrowed Bertrand Russell's characterisation of the epistemological foundations of the analytic sciences, mathematics, and accompanying analytic philosophy, as being based on higher order logic and a zero order, simple labelling type of semantics. The fundamental *modus operandi* is based on abstraction where all semantics and meanings of things are expressed within the rigorous confines of higher order symbolic logic and symbol dominated mathematics. The analytic paradigm transforms all problem domains into the world of abstract reason.

The applicability of abstract theory to practice is subject to the test of empirical methods. The author often imagines knowledge arranged like a two hemisphere epistemological brain where the analytic sciences were all lumped together in the left hemisphere and referred to as "left side sciences." Present day science is only half-brained. The problem is to develop a non-analytic but synthetic "right side science" to provide an alternative, complementary, much more holistic take on reality. Right side science was characterised as founded on zero order logic and higher order semantics.

The author argued that the Stoics thought along these lines with their logic of particulars playing the role of the zero order logic. Zero order logic means that there can be no variables ranging over sets and so abstraction becomes impossible. The downside of a complete loss of abstraction is counterbalanced by the upside of a rich semantics expressible both algebraically and geometrically. In this paper, we will develop this kind of science further and in greater depth.

Emmanuel Kant saw this left side, right side dichotomy in knowledge as that between *a posteriori* knowledge and *a priori* knowledge. *A posteriori* knowledge always needs *a priori* knowledge in order to become operational; For example, empirical sciences need to harvest empirical data before any abstract generalisations become possible. Axiomatic mathematics needs axioms before deducing theorems. The fundamental construct is that of an antecedent-consequent relation of some kind. In other words, the construct is *diachronic*, where the two

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<sup>1</sup> Completeness also applies to NAND, its dual, and called the *Sheffer stroke* after Sheffer's 1932 paper

sides of the epistemological equation are ordered in some way. The antecedent must precede the consequent and not the other way around.

In the right side science case of *a priori* knowledge, there can be no *a priori* to the *a priori*. In order to be operational, the science cannot seek assistance outside its own immediacy; it must rely on its own means here and *Now*. Such knowledge must be intrinsically operational right from the start and so can properly be termed as *operational science*. Operational science must rely on constructs where all players are immediately present in any whole. Instead of diachronic, the constructs for operational science must be *synchronic* in nature.

The Operational Calculus pioneered by Heaviside, provided a synthetic alternative to the analytical approach of ordinary calculus. Despite its synthetic, operational leanings, the Operational Calculus is still rooted in the analytical side of mathematics. Nevertheless, the Operational Calculus does provide a glimpse of what a purely synthetic science might look like. The most important principle underlying the pure synthetic approach is that of First Classness (FC), a term borrowed from Computer Science and already discussed in SON. FC plays an analogous normative role to that of axioms in analytic mathematics but not as an *a priori* construct. The FC principle is not an antecedent to its object; it is logically and temporally concurrent. As such, rather than being a logical principle, FC is *ontological*. The FC principle was discussed in some detail in SON.

FC is an extremely simple but profound principle and notoriously difficult to formalise, even in Computer Science. For our purposes, it can be simply taken as another name for the non-duality requirement. Any violation of non-duality is a violation of FC. Any system that satisfies FC must be non-dual; that is to say, the system must be free of any determined dichotomy. The most fundamental consequence of this non-duality constraint is that the whole system must be made of the same stuff. This can be seen in Computer Science where there are many examples of a limited form of FC. In each case, there is an accompanying mantra that “Everything is a something-or-other,” where the something-or-other refers to the stuff the system is made of.

The mantra for Object oriented systems is that “Everything is an Object,” accompanied by the explanation that ordinary objects, classes, meta classes and meta-meta classes, are all objects in their own right. In Category Theory, the mantra becomes “Everything is a morphism,” accompanied with the explanation that there are concrete morphisms, abstract morphisms, functors, and Natural Transformations. A common ingredient in all these examples of FC is that not only is the corresponding system made of the same stuff, but also there are four types of the stuff arranged in a Three-plus-One Structure, as first discussed in SON.

In the case of the Operational Calculus, the mantra becomes “Everything can be expressed in terms of polynomials of a complex variable.” This is the realm of practical engineering systems where there are clearly three different types of entity, notably the system, the input to the system and the corresponding output. The genius of the Operational Calculus is that all three of these types of entity can be represented in the same way in terms of polynomials of a complex variable. This is an example of FC. The other point to notice is that the polynomials form a multiplicative *algebra*. The diachronic form of knowledge that characterises analytic thought gives way to the synchronic form of knowledge expressed as a simple multiplicative algebra. In this case, differential equations give way to polynomials of a complex variable.

Of course, the fact is that not everything can be expressed in the form of polynomials of a complex variable and so Heaviside’s Operational Calculus is in no way universal. In order to arrive at the pure, universal version of the Operational Method, one must engage in a much more nuanced and deeper approach. In SON, the author identified Stoic philosophy as

containing the critical ingredients, when reconstituted with a bit of reverse engineering to fill in the missing gaps.

### 1.1 The Heaviside Operational Template

Heaviside's Operational Calculus provides a practical example of an operational alternative perspective to analytical methodology. We must repeat the endeavour, but on a much grander scale, right across two radically different epistemologies. Before moving on to the task ahead, we recap how Heaviside's work was accessed in the 1920's.

Heaviside's own work is not systematically arranged, and in places, its meaning is not very clear. Bromwich's discussion of his method by means of the theory of functions of a complex variable established its validity; and as a matter of practical convenience, there can be little doubt that the operational method is far the best for dealing with the class of problems concerned. It is often said that it will solve no problem that cannot be solved otherwise. Whether this is true would be difficult to say; but it is certain that in a very large class of cases the operational method will give the answer in a page when ordinary methods take five pages, and also that it gives the correct answer when the ordinary methods, through human fallibility, are liable to give a wrong one. (Jeffreys 1927)

Paul J Nahin recounts that in 1937, E J Berg sent Einstein a copy of his book *Heaviside's Operational Calculus*. Einstein wrote back a thankyou note saying that he appreciated the gift because *Now* he would be able to learn Heaviside's "peculiar mathematical witchcraft." (Nahin 2002) The task ahead is to develop operational methodology as a simpler and simplifying alternative to all of traditional science and mathematics. This is a tall order and sometimes we may be forced to do a bit of mathematical witchcraft along the way.

## 2 Analytic versus Synthetic Number

In this paper, the net is cast back even further than the Stoics, back to the Pythagoreans who embraced their own form of FC. The Pythagorean mantra was that everything could be accounted for in terms of number. As the famous quote goes, "The whole thing is a number."

Number amounts to the most fundamental entity in mathematics. Since we are interested in two kinds of mathematics, one analytic, and one synthetic, there must be two corresponding kinds of number. There must be analytic numbers, which will be based on a diachronic paradigm, and synthetic numbers that will be synchronic.

Examples of analytic numbers integers and reals. The diachronic structure of analytic numbers can be seen in the fact that the numbers are all ordered with respect to one another. Order is an abstract construct that enables numbers to formalise *quantity*. Analytic sciences are based on attributes that can be quantified using analytic numbers. A composite physical attribute can be represented by number tuples such as vectors. Vectors are *oriented* numbers.

From a right side perspective, the problem with analytic numbers is that they violate FC. An example of FC violation is the static, absolute dichotomy between negative and positive numbers. The only way around this problem is to use another kind of mathematics based on non-ordered numbers. This leads to *synthetic* numbers. Instead of being diachronic, synthetic numbers must be *synchronic*.

Synthetic numbers must be totally unordered. Imagine for a moment the kind of physics one could support using only synthetic numbers. The first casualty would be the conventional notion of time. Without quantified order, any notion of analysing the past and future goes out the window. All that is left is the *Now*. This is why we can refer to synchronic numbers as *now numbers*.

The development of a generic science based on *now numbers* will be driven by the one single requirement: avoid at all costs the introduction of a “before and after”, avoid introducing predetermined *a priori* or *a posteriori* constraints. The lack of predetermined *a priori* conditions is what Kant mandated for the science of his thing-in-itself. We refer to it as synthetic science, generic science, the universal operational science. The truth-normative and organisational principle of such a science will be FC.

### 2.1.1 Epistemological Gender

It follows that the lack of order means that synthetic numbers cannot be used to represent quantity. The only option left is that, in some way, synthetic number can represent *quality*. Qualities built from synthetic numbers would be totally different to the analytic based attributes. What is something that is not an attribute but nevertheless represents an aspect of the specificity of an entity? Every attributes represents a prison to the attributed. Granted, no entity is without specificity, but does it have to wear specificity as a lifelong, inescapable badge? An alternative to the attribute is the *disposition*. Attributes lead to a categorical approach to science. The alternative, non-categorical approach is dispositional. We won't attempt to fall into an analytic mode of discourse and try to define dispositions with a pin point definition and hence attempt to drag the dispositional back into the categorical land of attributes.

Unlike left side sciences, right side science is based on dispositions, not on attributes. Composite attributes ultimately are structures made up of simple analytic numbers. In an analogous way, composite dispositions are structures made up from simple elementary dispositions. The question is, what are the elementary dispositions and how many of them are there? This question was effectively answered in SON, where it was argued that there are only two elementary dispositions in the form of the two *epistemological genders*.

The epistemological gender construct was described as follows. An entity was said to be of pure *feminine* gender if it was totally devoid of any determined specificity whatsoever. Although not possessing a determined attribute, the feminine typed entity is endowed with a quality, notably that of being devoid of any specificity whatsoever. In order not to violate FC, the quality must be considered as an entity in its own right, otherwise there would be a determined dichotomy between entity and quality. The gender of this quality entity will be *masculine*. Thus, there are two universal elementary entities. The feminine entity has no other specificity than that it *has* a quality. The masculine entity *is* that quality. These are the two elementary dispositions, one feminine, and the other masculine. Gender is a very ancient construct *known* to thinkers in both the East and West. To our knowledge, gender has never been formalised in this more modern *has-a* and *is-a* way.

The pure feminine entity is different from the pure masculine entity because of a difference in gender. However, to any *third* party, the two entities are indistinguishable as any comparison of their qualities is impossible: there is only one quality between them; one entity has it the other is it.

Epistemological gender forms the basis of a universal typing system for a fundamentally non-dualist science. Knowing nothing *a priori* about a system or organism, but provided the mechanism be based on FC, one can say that it will consist of two gendered entities, one typed as F and the other as M. The two letters M and F can be thought of as the most elementary of all synthetic numbers. Henceforth, all synthetic numbers will be made up of composite structures of M and F.

### 2.1.2 The Four Synthetic Numbers

Each analytic science starts with some kind of determination concerning the entity under scrutiny, the object of study. The object of study for synthetic science is quite different; it is totally undetermined. The upside of the approach is that the resulting science will be unspecialised. In other words, the science will be universal and thus applicable to any entity whatsoever. The only requirement is that the entity does not violate non-duality and hence FC.

The object of study is what Kant called the *thing-in-itself* or what we call *any-entity-whatsoever*, or simply the *generic entity*. However, on focusing on this mysterious entity we find that it is necessarily endowed with a quality. Because of the FC requirement, the quality must be an entity in its own right. Thus, instead of one entity, we find that there are two, each of different epistemological gender. Somehow, the very act of even thinking about the generic entity has split it into two. The original pure generic entity has become feminine gendered and is *Now* accompanied by second entity of masculine gender.

This initial foray into the world of epistemologically gendered entities is quite admirable except for the fact that it is all founded on a fatal error. We want to study the generic entity pure. However, as a result of just thinking about it we modify our object of study to something else. The way out of this conundrum is to compensate for our initial error, but how?

The situation is like a lottery game. From the fairness rules of lottery, we *know* that one of the tickets in the urn has the winning number on it. This ticket is literally any-ticket-whatsoever as far as the game is concerned, because the game is not rigged and so accords to a certain kind of FC. Naively, we randomly choose any-number-whatsoever with the help of an automated Quick-Pick facility. The Quick-Pick facility is also fair and guaranteed to always come up with a random ticket. Our expectation is to win the lottery as any-ticket-whatsoever in the urn will have the same number as the any-ticket-whatsoever dealt out by Quick Pick – after all, both are any-ticket-whatsoever. A spoil sport might dampen our unbridled optimism by pointing out an error in our reasoning. The any-ticket-whatsoever in the urn might indeed be the winning ticket as stipulated in the fairness rules, but this ticket is not the same entity as the any-ticket-whatsoever that we have collected in hand. Moreover, even if we did by a freak chance actually win, the two tickets are different as a ticket before the draw has not the same value as a ticket after the draw.

However, we need not take this bucket of cold water lying down. We could object that this refutation is just another example of analytic, diachronic thinking that is so endemic in today's sciences. We are advocates of synthetic synchronous thinking. We could take up the position of the Stoics and argue counter intuitively that there is no such thing as blind chance. There is no luck; everything is fated. What we want is a world where the unchosen any-thing-whatsoever is effectively identical to the chosen any-thing-whatsoever. In this way, there can be no losers. Everyone wins the lottery. We want a world where the before and after are of no fundamental significance. What matter is the world of *Now*. Everyone is a winner *Now*. The prize for winning is nothing else but to objectively exist in the *Now*. Following Stoic reasoning, if you exist, you are a winner.

There are many ways of explaining the identity between the unchosen and the chosen generic entity other than this lottery allegory. Sankara, the early exponent of Advaita Vedanta, advanced the principle of Non-Duality explaining it in terms of indistinguishability between the individual soul atman and the cosmic soul Brahman.

The role of this paper is to provide a formalisation and demystification of the non-dualist construct. The entity of study for our science is the generic entity. So far, our analysis has revealed that not only does the generic entity come in two parts but also there are two kinds

of generic entity, one chosen and one unchosen. One is the generic entity *somewhere*, the other *anywhere*. Both will consist of two opposite gendered parts. The gender typing of the unchosen *anywhere* generic entity will be denoted by the uppercase letters F and M. The letters for the chosen *somewhere* entity will lowercase. Our approach is operational, hence synchronic, and algebraic. The idea is that the both kinds of generic entity are present in the same instance; moreover, they somehow make up a harmonious whole. The two generic entities must be identical in some way and hence avoid violating the principle of non-duality, that is to say, the principle of FC.

As indicated in previous work, the algebraic formulation of this scenario can be expressed by the product of the two generic entities as follows:

$$\begin{Bmatrix} m \\ f \end{Bmatrix} \otimes \{F \quad M\} = \begin{Bmatrix} mF & mM \\ fF & fM \end{Bmatrix} \quad (1)$$

The simply typed M and F entities are not tractable in isolation as there is the ambiguity concerning the context. The question naturally becomes, is the context that of the *somewhere* generic entity or the *anywhere* generic entity? In (1), there is no ambiguity as both entities participate in a unified whole and four binary gender types result.

The non-dualist result is a single unified generic entity that, instead of being made up of two differently gendered entities, *Now* has four. The lower and uppercase lettering is *Now* superfluous as the information is implicit in the ordering of binary gender. From the above algebraic formulation of this age-old problem, we find that there are *Now* four entities MF, FF, FM, and MM that play the role of elementary synchronic numbers, the four base *now* numbers.

Empedocoles considered the four types as the four roots making up reality leading to the four classical elements Air, Earth, Water, and Fire respectively. The Stoics employed a like formulation with our lower case letters *m* and *f* interpreted as *active* and *passive* modes. To them, the classical elements Air, Earth, Water, and Fire were typed *active F*, *passive F*, *passive M*, and *active M* respectively.

The square, 2x2 “matrix” that appears on the right side of (1) can be interpreted as a *semiotic square* illustrating the fact that any whole can be divided up into four component modes. The semiotic square is a universal structure and is an alternative to Aristotle’s Square of Oppositions, as discussed in SON. In modern times, Algirdas Greimas introduced his version of the semiotic square notion, leveraging it off Aristotle’s square. Greimas pioneered the empirical semiotic analysis of a wide range of discourse from the Bible, literary works, to art criticism. According to Greimas, the semiotic analysis of a well-written novel like Maupassant, would be analysable into a sequence of semiotic squares where one semiotic square “changes gears” into another (Greimas 1991).

As one can see, the four-element explanation of reality has a long pedigree.

### 3 Scalar and Non-Scalar Complex Number

In this paper, the emphasis is on the four binary gender types considered as synthetic numbers. When trying to understand entities that make up synthetic mathematics there will always be some kind of analytic version in traditional analytic mathematics that may be of conceptual assistance. To begin with, consider analytic numbers from a qualitative point of view. The most characteristic feature of an analytic number is that it is a scalar. Scalars are the perfect expression of zero-order semantics as each individual number is nothing more than a label for some particular quantity. The analytic number has no other intrinsic semantic implication than its numerical value. Analytic mathematics is more interested in the logic of things, particularly the logical implications of numerical value, and the implicit ordering. The resulting analytic, forcibly abstract, theory of number can be very complex, challenging, and

sometimes very elegant. However, the semantics will still remain zero-order throughout, as indeed it must. This is just the way that Bertrand Russell would have wanted it. There is nothing wrong with the analytic methodology. Our objective is to complete the picture by introducing an alternative, complementary methodology.

### 3.1.1 The Non-Scalar Entity is Geometric

Based on zero order logic but high order semantics, synthetic mathematics has the opposite epistemological structure to its analytic counterpart. This is where the difference between analytic and synthetic numbers starts to become more evident. If analytic numbers are scalars, synthetic numbers are *geometric* entities. These generic entities are not abstract but instead they involve *universal generic* geometrical structures. The elementary basis elements of the generic geometries are the four binary gendered numbers MF, FF, FM, and MM. These numbers are not abstract generalisations of quantity, as is the case for analytic numbers. Instead, the four gendered types are universal, generic entities and can be interpreted geometrically.

Figure 1 illustrates a simple geometric interpretation of the four generic basis elements as different kinds of dyads. The four dyads differ by whether one or other of the endpoints is fixed or free. A fixed end of a dyad will be typed masculine; a free end will be typed feminine.

Another interpretation would be to interpret the four diagrams as representing cones and bunches of arrows so that the source and target ends of resulting dyad have different cardinalities. This is the One-Many interpretation. The interpretation can be useful in understanding but, in the final count, is too simplistic. The implication is that masculine gender means a cardinality of One and the feminine a cardinality of Many. The problem is that the cardinalities viewed in this way are quantities, not qualities; they are analytic numbers and thus violate FC. Nevertheless, the Many-One interpretation can be useful as it is easy to understand, although ultimately naïve and even misleading.

The distinction between analytic number and synthetic number could be interpreted as a distinction between ordinality and cardinality. In his book *Number*, Midhat J. Gazalé discusses the two primary aspects of number as a distinction between ordinality and cardinality where the latter is about naming, telling things apart, without regard to any necessary ordering. In other words, ordinality is intrinsically quantitative whilst cardinality is intrinsically qualitative. However, the cardinality mentioned in this sense is not compatible with the cardinal numbers of Cantor's Set Theory. Set Theory is anchored in the analytic side of things and so set theoretic cardinal numbers become subverted and can be arranged in a pecking order. The cardinality of cardinal numbers opens the door to the One, the Many, and the number of all the subsets of the Many – the Many-Many, which is greater than the first two cardinalities. For cardinality to play the qualitative role imagined by Gazalé there must be a clean break from ordinality. The purely generic cardinal numbers must not be capable of being ordered. To do so would be to violate FC. Instead of cardinality in terms of the One and the Many, the generic form must speak qualitatively in terms of *Oneness* and *Manyness*. There are just as many instances of Oneness as there are of Manyness. To wax lyrical, any Many is One and any One is Many. In this context, the terms Oneness and Manyness would be interchangeable with the terms masculine gender and feminine gender.

SON also discussed a spacetime geometry interpretation of the four binary genders. The MF typed geometric entity was interpreted as a cone of timelike lines, FM as spacelike cone, and FF as a bundle of lightlike lines. Thus, a conventional spacetime diagram for a two dimensional Minkowski space can be built from an MF, FF, and FM typed dyad. The paper added in a fourth component that is missing from conventional spacetime geometry, the MM

dyad that corresponds to a *singular* line. The resulting spacetime geometry thus provided a geometric interpretation

The spacetime geometry interpretation would indicate that the four generic bases behave like imaginary numbers where the MF lightlike line has a positive square, the FM spacelike line a negative square, the FF lightlike line has a zero square. Thus, it would appear that the four elementary kinds of synthetic number behave something like the real and imaginary numbers of ordinary analytic mathematics.

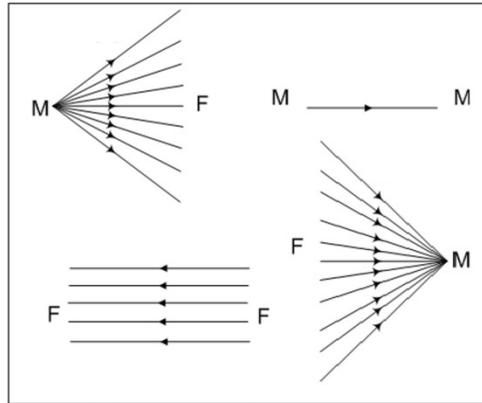


Figure 1 Geometric interpretation of the four generic bases as cones and bundles of arrows.

### 3.2 Analytic Imaginary Numbers

In this section, we briefly review ordinary real, imaginary, and complex numbers from a slightly skewed perspective by deliberately leaning to the right side way of thinking. The usual analytic approach is to extend analytic numbers to an algebraically complete system. The classic problem was that not all solutions to a quadratic equation are an analytic real number. To solve the problem, imaginary numbers were invented. The legendary unit imaginary number was defined as the square root of  $-1$ , usually denoted by the letter  $i$ . The two solutions for a quadratic was a complex number of the form

$$x = \alpha \pm \beta i \text{ where } \alpha \text{ and } \beta \text{ are analytic numbers} \quad (2)$$

Thus, for completeness, the true basis for analytic computation is not a single type of number but a binary compound made up of two types of number, one said to be real and one imaginary.

Amazingly, when it comes to conventional applied mathematics and engineering, one can make do by simply using ordinary complex numbers. In most practical applications, no other number types are necessary. However, although not strictly necessary for applied mathematics, it is well known that there are other kinds of complex number than just the ordinary version. This leads us to an investigation into *hypercomplex* numbers. Hypercomplex numbers are still made up of scalar valued analytic numbers; however, they do provide a pale shadow of their generic counterparts on the right side of the epistemological divide and so are worth reviewing. We proceed as follows.

#### 3.2.1 The Four Kinds of Complex Analytic Number

Examining the theory of simple quadratics reveals that there are three types of imaginary number as shown by looking at the solutions for the quadratic  $Q(x)$  defined by:

$$Q(x) = x^2 + ax + b = 0 \quad (3)$$

There are three cases where  $x$  is a pure imaginary number.

**Case 1:**  $a=0, b=1$

In this case, the solution for  $x$  becomes the *ordinary imaginary* number  $i$  defined by:

$$i^2 = -1$$

which, when combined with a real number, becomes the *ordinary complex number* of the form:

$$x = \alpha \pm \beta i$$

**Case 2:**  $a=1, b=0$

In this case, the solution for  $x$  becomes the imaginary number  $j$  defined by:

$$j^2 = +1$$

which, when combined with a real number, becomes the *split complex number* of the form:

$$x = \alpha \pm \beta j$$

**Case 3:**  $a=0, b=0$

$$k^2 = 0$$

In this case, the solution for  $x$  becomes the imaginary number  $k$  defined by:

$$k^2 = 0$$

which, when combined with a real number, becomes the *dual complex number* of the form:

$$x = \alpha \pm \beta k$$

**Case 4:**  $a \neq 0, b \neq 0$

From the traditional analytic, left side point of view, this case, and the way we handle it, might look a little odd. However, just for fun we soldier on with our right side biased perspective and should come up with a fourth type of number – for the right brained thinker, everything seems to come in fours, or at least Three-plus-One structures. We will refer to the fourth type of number as a *real* number and denote it by  $z$ . An analytic mathematical equation for  $z$  would be

$$z^2 = Q(x) = x^2 + ax + b \tag{4}$$

Thus,  $z$  represents the *whole* quadratic  $Q(x)$ . Intuitively, one would reasonably expect that  $z$  could be expressed as the linear combination of the three imaginary numbers  $i, j, k$ , as follows:

$$z = \alpha i + \beta j + \chi k \tag{5}$$

However,  $j, k$ , and  $i$  are *diachronically* related, not synchronically. They cannot appear synchronously in one single algebraic equation like (5) as they are defined by mutually exclusive cases 1, 2, and 3 as previously discussed. In brief, a solution for (4) in the form of (5) is impossible using only analytic scalar numbers.

This was this kind of problem that faced Hermann Grassmann in the nineteenth century. He resolved the problem by inventing a new kind of number that he referred to as *oriented number*. After Grassmann, the oriented number concept took two different historic paths, one algebraic and one based on coordinates. The algebraic approach is more on line with the general thrust of this paper and will be discussed further on. The coordinate approach leads to vectors and was pioneered by Heaviside and independently by Gibbs. The vector approach introduces a higher order kind of analytic number in the form of  $n$ -tuples of elementary analytic numbers. Using this approach for expression (5), the imaginary numbers  $i, j$ , and  $k$  are replaced by basis vectors  $\mathbf{i}, \mathbf{j}$ , and  $\mathbf{k}$  which squares of  $-1, +1$ , and  $0$ , respectively. Vector analysis leads to analytic forms of geometric spaces. The general form of a vector space is usually denoted by its signature  $(p, q, r)$  where  $p$  is the number of positive squares,  $q$  the number of negative, and  $r$  the number that square to zero. Vector spaces that include basis vectors that square to zero are generally considered redundant and so  $r$  is usually assumed zero in traditional geometries.

In most practical applications of left side mathematics, only the ordinary imaginary number  $i$  is considered. As for the imaginary number  $j$ , it is often treated as a real number. Overall, one can say that the left side mathematical paradigm is not particularly adept at exploiting the potentials of hyper-complex numbers. As we shall see, the right side paradigm will take a different approach to the four  $i, j, k$ , and  $z$  mathematical entities. Instead of

interpreting them as scalars, these four types will be considered as *geometric* entities – *generic* geometric entities.

Nevertheless, left side mathematics is not devoid of geometric constructs. A simple example is the polar representation of hyper-imaginary numbers in the form  $e^{\theta m}$  where  $m$  can be either  $i, j$  or  $k$ . In the case of ordinary imaginary number  $i$ , the polar form, together with its conjugates, can be used to define trigonometric functions where:

$$2\cos\theta = e^{\theta i} + e^{-\theta i} \quad \text{and} \quad 2i\sin\theta = e^{\theta i} - e^{-\theta i}$$

The dual number  $j$  can be used to define the hyperbolic functions:

$$2\cosh\theta = e^{\theta j} + e^{-\theta j} \quad \text{and} \quad 2j\sin\theta = e^{\theta j} - e^{-\theta j}$$

For the want of a better name, we can call the corresponding functions defined by  $k$  type imaginary number  $k$  the *nullametric* functions. We name them *sinn* and *cosn* respectively as defined by:

$$2\cosn\theta = e^{\theta k} + e^{-\theta k} \quad \text{and} \quad 2ksinn\theta = e^{\theta k} - e^{-\theta k}$$

Since  $e^x = 1 + x + \frac{x^2}{2!} + \dots$

substituting  $x = \theta k$  and exploiting that  $k^2 = 0$  then

$$e^{\theta k} = 1 + \theta k \quad \text{and} \quad e^{-\theta k} = 1 - \theta k$$

which illustrates that the polar and Cartesian forms are the same for dual complex numbers.

The nullametric functions simplify to

$$\cosn\theta = 1 \quad \text{and} \quad sinn\theta = \theta$$

Left side mathematics tends to treat  $k$  type numbers but the above does show that they are not as pathological as one might think. In most practical applications left side mathematics only deals with ordinary imaginary numbers. As far as the real number  $r$  is concerned,  $r$  is not distinguishable from the double number  $j$  as they both have the same square. From a right side perspective, any left side attempt at a complete theory of hyper-complex numbers will necessarily be fragmentary and incomplete. In the right side approach, the four kinds of number  $i, j, k$ , and  $z$ , are replaced by four elementary geometric forms labelled as *fm, mf, ff*, and *mm* respectively. As we shall show, it is only in the richness of a generic geometry and not in the poverty of left side scalar entities that a complete theory of hyper-complex entities becomes possible.

#### 4 Synthetic Numbers and the Stoics

Our task is to develop mathematics based on a system of synchronic numbers rather than the traditional analytic number approach. Synchronic numbers are qualitative rather than quantitative. The aim is to develop a universal algebra and geometry founded on the qualitative *now numbers*. The SON traced the beginnings of the universal algebra to the four-letter alphabet of Empedocles' theory of the four classical elements, through Aristotle's Syllogistic Logic, and onto Stoic Physics and Logic. The paper added the beginnings of a geometrical interpretation that will be further developed in this paper, finally leading to a universal version of the Geometric Algebra pioneered by Grassmann, Hamilton, Clifford, and in modern times, David Hestenes. The result is a harmonious synthesis of logic, geometry, and algebra proposed as expounding the hidden semantics of the four-letter universal generic code of Nature.

In SON, the author sketched out how Stoic philosophy provides a ready-made template for the foundations. The Stoics taught their philosophy in three parts of an integrated whole consisting of Ethics, Physics and Logic. For the Stoics, the ethical principle must encompass not only the morals of what is good and bad for the organism, but must also play a normative role in physics and logic. Our version of the ethical principle is that embodied in the principle of FC. One example of FC in Stoic thinking on physics was that they considered an attribute

of an entity to be an entity in its own right. There can be no second-class citizens according to FC. Stoic Physics was founded on a five-element theory incorporating the four classical elements with the universal pneuma as the fifth element. In SON, the author sketched out how the five indemonstrable syllogisms of Stoic logic match up directly to the five elements theory of their physics. The result accomplished by the Stoics was an incredibly tight and unified system. However, there is one missing ingredient from their system – mathematics. By mathematics, we mean the unity of three fundamental disciplines, notably number, algebra, and geometry. In this section, we revisit the relationship between Stoic logic and Physics, but this time the emphasis is on the mathematics of the universal, synthetic science that we are developing.

#### 4.1 Synthetic Imaginary Numbers, a Stoic Contribution

The fundamental thrust of this paper is that the gender construct provides the basic four foundation stones for synthetic science and its mathematics. This can be seen most clearly in the relationship between gender and imaginary numbers. For example, the binary gender type *fm* was interpreted above as the imaginary number *i*, the infamous “square root of -1.” Gender uncovers the finer structure involved with number, integrating the qualitative with the quantitative. The relationship between gender, imaginary numbers, and geometry can be made more formal through the insights provided by Stoic logic.

The point of departure for the generic geometry that we need is Geometric Algebra (GA). GA is based on the Clifford Algebras and is sometimes described as geometry founded on quadratic forms. A number of important concepts can be raised about quadratics that are pertinent for the task ahead. Consider the simple quadratic form  $Q(x) = 0$ , i.e.

$$x^2 - px + q = 0 \quad \text{where } p \text{ and } q \text{ are real} \quad (6)$$

Real linear equations only have one solution, a scalar real value. A typical solution for the quadratic will be a pair of complex numbers. Thus, the solution of the quadratic is not a scalar, but becomes a pair of points that can be plotted on an Argand diagram.

The solution has a geometric flavour. A geometric flavour and geometric representation shouts out semantics. Informally, we can say that the solution to a quadratic equation evokes a richer semantics than the simple linear equation. We might be tempted to call the semantics first order semantics to distinguish it from the zero order semantics of the linear form. However, we are jumping the gun. Solving a quadratic is still bread and butter traditional left side mathematics and we have already pigeon-holed all of the left side sciences as having zero order semantics. We incorrectly said that a linear form is in one-to-one correspondence with its single solution when really the relationship is one to many. Fundamental to all left side sciences is the abstract notion of variable ranging over sets: the variable  $x$  can have any value in the set  $C$  of complex numbers. The ordinary vanilla quadratic might look as if it is a non-diachronic structure and hence synchronic – after all, it has two simultaneous solutions. However, the whole construct is still tainted through and through with abstract variables and abstract symbols ranging over sets of values. The quadratic is decidedly a diachronic structure and not synchronic and hence has zero order semantics. We might grant it first order *abstract* semantics, or even first order *pseudo*-semantics, but that is all.

In order to achieve true higher order semantics we must first rewrite the quadratic as

$$Q(\mathbf{X}) = \mathbf{X}^2 - p\mathbf{X} + q = 0 \quad (7)$$

where  $\mathbf{X}$  is a geometric entity and thus not a variable with values ranging over a set. In what follows, the geometric entity designated by  $\mathbf{X}$  will turn out to be binary gender typed.

In line with our very geometric and right handed approach, the quadratic can be separated into two parts based on the natural opposition between the even powers of  $\mathbf{X}$  and the odd power. Putting the odd on the left and the even on the right side of the equation, we get:

$$pX = X^2 + q \tag{8}$$

This is a convenient left-side, right-side way to express the quadratic. We will *Now* interpret this quadratic form as some kind of generic substance with linear and bi-linear characteristics. However, we are not modern mathematicians but thinkers from another time – a time well before abstract thinking became the prevalent form of reasoning. In fact, our thinking is so ancient that when confronted with this generic substance, we immediately turn our mind to figuring out how the substance is constituted from the four classical elements.

Not being abstract thinkers, we think of this quadratic substance, not from a general, abstract perspective but from a non-abstract, *universal* perspective. If it is a universal form then it must be subject to the dictates of FC. As such, it cannot be a diachronic structure but must be synchronic - anchored in the immediacy of *Now*. This substance can be considered from the point of view of matter-as-rationality or from the viewpoint of rationality-as-matter. Since no abstraction is allowed, the logic involved in the rationality must but be based on zero order logic – the logic of particulars. Such logic is naturally ontologic in nature, as only particulars can exist, abstractions cannot. We turn to the logic of the Stoics based on four of the five indemonstrables. Stoic logic is zero order.

The first task is to investigate this quadratic substance from the point of view of appearances. The appearance of the substance can be known by the presence or absence of attribute. The attributes of appearance we call imaginary, as they are only a means of knowing, perceiving, or comprehending a thing; we will refer to them as dispositions. Dispositions pertaining to appearance will be qualified as being *imaginary*, and are not *real* things. Dispositions are in fact synthetic numbers. Just as the analytic numbers of left side mathematics can be real as well as imaginary, the same applies to synthetic numbers. Thus, there can be real dispositions. However, real dispositions are not the real thing either. They are what the Stoics called *lekta* or “sayables.” Sayables express how things can be known. Knowledge of things can be expressed in terms of complex dispositions having real and imaginary parts. Thus, dispositions, as universal sayables, explain the nature of things and the how and why of appearances. Universal dispositions are synthetic numbers or more colloquially, *now numbers*.

Note that by appearance, we mean how an entity appears *to itself*, not to a third party. From the perspective of left side science, appearance is expressed in terms of attributes perceivable by a third party. Right side science is concerned with how the entity appears to itself, and that cannot be expressed in terms of attributes, only via dispositions. An entity can know itself via its real and imaginary dispositions. Dispositions are constituted from elementary elements that are universal, applying to any organism that is capable of maintaining the coherence of non-duality as demanded by FC.

The first three dispositions of generic appearance can be studied in terms of the Stoic second, fourth and fifth indemonstrables. These three dispositions are imaginary and combine to determine a real dispositions which can be studied via the first indemonstrable. The overall coherence of any such system is dictated by the third Stoic indemonstrable, which expresses the generic version of the law of non-contradiction. The third indemonstrable applies to all four cases. The details of the four cases are as follows.

#### 4.1.1 Application of Stoic Logic to the Semantics of the Quadratic

Here we present an alternative perspective on quadratic forms. We are going to look at a finer, more subtle structure than the approach of traditional mathematics. Our objective is to reveal the semantic-cum-semiotic structure of quadratic forms. This finer structure can then be exploited to provide a basis for a universal form of geometry.

The quadratic has two qualities corresponding to  $p$  and  $q$  in (8). In applying Stoic Logical forms to the quadratic, it might be tempting to interpret  $p$  and  $q$  as the “first” and “second”

qualities that appear in each Stoic syllogism. However, the logic is more subtle than that – more dialectic. What appears as first in one context may appear as second in another. The notion of context is formalised in the Stoic syllogistic system. There are five fundamental contexts, one for each indemonstrable. As first explored in SON, each syllogism, other than the third, can be illustrated by a Heraclitus diagram that represents the rational orientation in the logic, formalising the “Heraclitus flux” so to speak. The left and right circles of the Heraclitus diagram represent the first and the second qualities respectively, referred to in the major premise of the syllogism.

The minor premise of the syllogism refers to one of the qualities mentioned in the syllogism, either the first or the second. The minor is represented on the diagram by the source of the diagram’s dyad. The orientation of the dyad may go from left to right or right to left, depending on the context, i.e., depending on the syllogism. The source of the arrow dyad always represents the quality  $p$ . Depending on the syllogism, the source  $p$  of the dyad may be located either on the left or on the right. The source  $p$  of the dyad represents a quality that may or may not be present. If present, it will be of masculine gender. If not present, it will be feminine. When appearing in a mathematical equation, the masculine leads to a value of  $p=1$ , the feminine to a value of  $q=0$ .

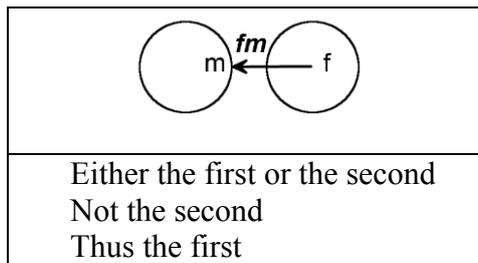
Implication flows from the minor premise to the concluding premise. In a Heraclitus diagram, the implication is represented by an arrow dyad with source  $p$  and head  $q$ . The head will appear on the opposite side of the diagram to the source. The quality referred to in the concluding premise will correspond to  $q$ . Like  $p$ , the  $q$  quality is gendered either masculine or feminine and in the quadratic equation will have a value of 1 or 0 respectively.

Note that for the four syllogisms considered; there are two forms of the major premise. The fourth and fifth syllogism major premises are disjunctive propositions and so place no explicit ordering on the first and second qualities. The ordering semantics appear in the minor and concluding premises. In both cases, the gender of the source  $p$  is the opposite to the gender of the head  $q$  and the direction of the arrow dyad – left to right or right to left - does matter. In the case of the first and second syllogisms, the gender of the source  $p$  and head  $q$  are the same – either both masculine, or both feminine. In these two cases, if the major premise were a disjunctive, and hence semantically symmetric and interchangeable for the first and second quality, then the semantics would also be symmetric in  $p$  and  $q$ . In this case, the direction of the arrow dyad would be semantically meaningless. The first and second syllogisms would be degenerate statements with tautological semantics. To avoid this degenerate situation, the premises of the Stoic first and second syllogisms are expressed with the directional semantics of the conditional – “If the First then the Second.” The first and the second qualities become *logically* first and second. The arrow dyads in the diagram for each of the first two Stoic syllogisms *Now* have a logical reference and thus have non-null semantic clout. By using the conditional form for the major premise, the first and second syllogisms become non-degenerate. Thus, finally, the orientation of *mm* and *ff* gender typed dyads does matter – despite their gendered symmetry.

The five indemonstrables form a very tight system. Although the gist of Stoic logic would be due to Zeno, the beauty and elegance of the five indemonstrables would be surely due to Chrysippus. We *Now* consider how the five indemonstrables can articulate the logical structure of the quadratic form in each of its contexts. The third indemonstrable does not correspond to a determined context, has no Heraclitus diagram, and so is not included.

**Case *fm*** - the fifth indemonstrable:

We start with the fifth syllogism as it leads to an analogous construct familiar in traditional mathematics.



**Figure 2** The fifth indemonstrable syllogism and its Heraclitus diagram.

The first and the second qualities referred to in the major premise in the fifth syllogism are represented by the left and right circles respectively in the Heraclitus diagram. This establishes the convention that the positive direction is from left to right. The quality referred to in the minor premise is the second quality. Thus, according to the preceding rules, the source  $p$  of the arrow dyad is on the left side of the diagram. The minor states that the second quality is not in possession and thus the source  $p$  of the arrow will be typed feminine. The head  $q$  of the arrow will be on the other side of the diagram, the side of the first quality. The concluding proposition declares that the first quality is in possession and thus will be gender typed as masculine. In summary, the Heraclitus diagram for the fifth syllogism will be represented by an dyad oriented in a negative orientation going from right to left and will be typed *fm*. This binary type *fm* is the dispositions defined by the fifth syllogism.

Turning *Now* to the corresponding quadratic form, the quantitative value for the feminine typed  $p$  will be zero and the value for the masculine typed  $q$  will be one. The quadratic in this context reduces to:

$$0 = X^2 + q \tag{9}$$

thus

$$X^2 = -1 \tag{10}$$

In other words, the geometric object  $X$ , whatever it is, squares to  $-1$ . The result can be interpreted two ways. Firstly, the synthetic number,  $X$  squares to the analytic number  $-1$ . Alternatively, to avoid any implicit ordinality, the  $-1$  entity could be interpreted as a synthetic number expressing the semantics of “contraction” or negative divergence as implicit in the Heraclitus diagram. The contraction semantics is quite generic and can be thought of as a contraction of degrees of freedom.

Possibly, Heraclitus somehow intuitively thought along these lines in terms of some kind of “Heraclitus flux.” It would be interesting to know how Chrysippus interpreted the indemonstrables in the context of Stoic Physics, but it looks like as if we will never know. The claim the author makes is that Chrysippus would have interpreted the flow of rationality in the indemonstrables as a Heraclitus kind of flux. Bolt the building blocks together and one would end up with a complete system of interacting fluxes of rationality. Alternatively, take it all apart and one should end up with the four classical elements expressed as Stoic syllogisms. This is an interesting way of thinking, even though it might not be an historically accurate reconstruction of how the Stoics actually thought on the matter. In the meantime, the author asks whether anyone else of modern times has any better-detailed explanation regarding the Stoic claim that the philosophy of Heraclitus, Stoic Logic, Stoic Physics, and Ethics could all be integrated into the one unified science. It seems that existing scholarship has very little of substance to say on the matter.

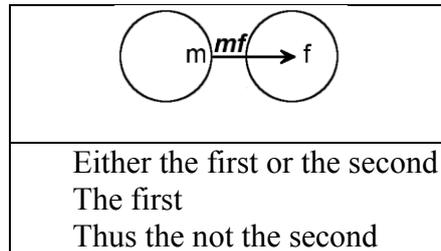
*Now* the question: what is this geometric object  $X$ ? Remember that  $X$  is not a variable ranging over some set of values. Variables require abstraction and we are after a universal result not a general abstract one. According to the right side science account, universal constructs are articulated in terms of gender. In short, from a universal perspective  $X$  should correspond to a gender typed geometric entity. Already, the Heraclitus diagram in Figure 2 illustrates such a gender typed geometric entity. It illustrates an *fm* typed arrow dyad oriented

in the opposite direction to the convention established in the major premise. Intuitively, we could say that the ***fm*** typed dyad shown in the Heraclitus diagram “has negative square”; it should square to -1. From such considerations, we have no further hesitation but to remove any suspicion that ***X*** is a variable and simply interpret it as a gender typed geometric entity in the form of an ***fm*** typed dyad with negative square. Thus, we can replace (10) with:

$$(\mathbf{fm})^2 = -1 \quad (11)$$

Thus, the geometric gender typed entity ***fm*** behaves like the ordinary imaginary number ***i*** from traditional mathematics, the imaginary number that squares to -1. However, in this case, ***i*** is not a scalar but a geometric entity with internal structure. The ***fm*** dyad is made up from the product of ***f*** and ***m***. This oriented dyad ***fm***, will form one of the bases of the universal algebra we seek. The product will be interpreted as a generic version of the geometric product of Clifford Algebra.

**Case *mf***: the fourth Indemonstrable:



**Figure 3** The fourth indemonstrable syllogism and its Heraclitus diagram

In the Heraclitus diagram for of the fourth indemonstrable, the major premise establishes the convention of the first quality on the left and the second on the right, the same as for the fifth indemonstrable. This time, the minor premise refers to the first quality and hence the source ***p*** of the dyad arrow will be on the left of the diagram. The minor premise states that the first quality is in possession and hence will be typed masculine. The head of the dyad will be on the right side of the diagram and so relates to the second quality. The concluding proposition declares that the second quality is not in possession and hence will be typed feminine. Overall, the Heraclitus diagram for the fourth indemonstrable represents an arrow dyad oriented from left to right, hence of positive square, and gender typed as ***mf***. The quadratic form in this case will have  $p=1$  and  $q=0$ , reducing to:

$$\mathbf{X} = \mathbf{X}^2 \quad \text{and so} \quad \mathbf{X}^2 = \mathbf{X} \quad (12)$$

replacing ***X*** with the geometric entity ***mf*** gives

$$(\mathbf{mf})^2 = \mathbf{mf} \quad (13)$$

Thus, ***mf*** squares to itself and so has the similar semantics as the unit scalar, which also squares to itself. Just as the ordinary imaginary number ***i*** square to -1, and the imaginary number ***j*** squares to +1. In both cases, ***i*** and ***j*** are considered as scalars in the perspective of left side analytic mathematics. From the *right* side perspective the universal generic version of ***i*** and ***j*** are the geometric entities ***fm*** and ***mf***. Thus, one could write:

$$(\mathbf{mf})^2 = \mathbf{mf} = \mathbf{j} = \mathbf{1} \quad (14)$$

where, this time ***j*** and its equivalent representation as a geometric unit ***1***, correspond to a geometric entity – an ***mf*** gendered dyad, to be precise. This ***mf*** dyad is another base element for a universal geometry.

Once again, the right hand side of the equation, the ***1*** entity, could be interpreted as the unit analytic scalar. Alternatively, from a synthetic number perspective, the ***mf*** dyad has the semantics of expansion, as illustrated in the Heraclitus diagram in Figure 3. It has positive divergence. It encompasses the semantics of the individual confronted simultaneously with the reality of its Oneness and the Manyness of its potential. In the context of spacetime geometry, the ***mf*** dyad takes on the semantics of a cone of timelike lines, for example.

Case **ff**- the second Indemonstrable:

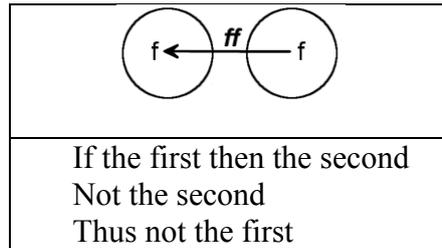


Figure 4 The second indemonstrable syllogism and its Heraclitus diagram

In the case of the fourth and fifth indemonstrables, the major premise did not distinguish semantically between the first and the second qualities. The only issue in play was that one quality was designated first and one second. This is purely a case of establishing a labelling convention. Reverse the labelling and there is no difference in semantics, only a reversal of the convention. What matters is the convention, not who or what ends up on one side, or the other. In the case of the second (and the first) syllogism, the major premise changes from establishing a semantically symmetric convention to locking in a semantic asymmetry. The asymmetry is expressed in the form of a conditional – “if the first then the second.” The fact that the first implies the second in no way justifies the reverse implication. The conditional is an asymmetrical construct. In the present context, the major premise not only establishes a convention concerning which is first and which is second, but adds a semantic logical flavour of the necessity arising from logical implication. The importance and subtlety of this play between symmetrical and asymmetrical constructs cannot be over-emphasized. This is the work of a master logician, almost certainly Chrysippus.

The logic of the major premise involves an implication oriented from left to right. The implication is not semantically neutral. The logic of the minor premise establishes that the source of the dyad is on the right side of the diagram; hence, the head will be on the left. The resulting dyad and its implicit implication will be in the opposite sense to that of the major premise. On its own, this dyad seems almost devoid of semantic content. According to the minor premise, the second quality is not in possession and so will be gendered feminine. The same applies to the head of the dyad referred to in the concluding proposition. The first quality is not in possession either and so will also be typed feminine. The result is that the Heraclitus diagram representation of the second syllogism will be in the form of an **ff** dyad oriented from right to left. On its own, the **ff** dyad is devoid of semantics. It simply becomes an implication that **f** implies **f**. This could just as well be **x** implies **x**, an intractable tautology. *Now*, the only real claim to semantic glory for the **ff** dyad is that it has an orientation from right to left. This orientation from right to left is the only determined specificity in play for the **ff** dyad. This is the only specificity that distinguishes the **ff** dyad from the semantic abyss of the tautological. *Now*, if the major premise were to be the same as that for the fourth and fifth syllogisms, the construct would become totally degenerate, as the right to left orientation of the **ff** dyad would be semantically insignificant. The **ff** dyad is oriented from right to left which is the opposite direction established by the premise. If the premise is merely an arbitrary convention with no objective logical order, such as the semantically symmetrical disjunctive, the construct becomes degenerate and meaningless. However, since the present context employs the conditional as major premise with an implication oriented from left to right, an implication oriented in the *opposite* direction actually must “mean something.” The construct based on the second syllogism escapes from the abyss of the meaningless tautology and becomes a non-degenerate construct – a geometric construct.

The corresponding quadratic form for the **ff** dyad will be the case where  $p=0$  and  $q=0$ , reducing to:

$$X^2 = 0 \text{ and so } (ff)^2 = 0 \quad (15)$$

The **ff** dyad is analogous to the imaginary number **k**, *which* also squares to zero. The **ff** dyad, constructed in this way in accordance to the second Stoic indemonstrable, can be used as a basis of a universal geometrical form.

Case **mm**: the first Indemonstrable:

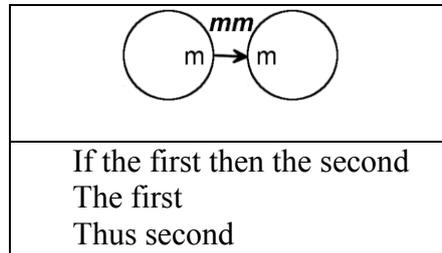


Figure 5 The first indemonstrable syllogism and its Heraclitus diagram

The three dyads **fm**, **mf**, and **ff** constructed from the fifth, fourth and second Stoic indemonstrables can be thought of as the three “imaginary” entities. These are the three limit cases. We *Now* come to the case of the “real” entity of type **mm**, that corresponds to the first Stoic syllogism. The major premise is a conditional establishing the reference left-right orientation for the Heraclitus diagram. The minor premise refers to the first quality and so the source p of the arrow dyad will be on the left side of the diagram. The first quality is in possession and so p will be of masculine gender. The head q of the arrow will be on the second quality side. The second quality is also in possession and so q will be of masculine gender.

Unlike the previous three “imaginary” cases where the form was broken down into its three generic sub-forms, this time the form is taken as a whole. This is why we are calling it the *real* case as distinct from the three *imaginary* limit cases. This time we have the first and the second quality.

In corresponding quadratic form will have both  $p=1$  and  $q=1$ . Thus:

$$X = X^2 + 1 \tag{16}$$

Substituting **mm** for **X**

$$mm = (mm)^2 + 1 \tag{17}$$

Informally, the doubly masculine **mm** dyad has no determined specificity except that it is an attribute *as* attribute or, more precisely, dispositions *as* dispositions. The double masculine is the most enigmatic of the four dyads and the most difficult to understand. A poet might say that **mm** represents pure Oneness as Oneness, but that does not help in formalising the concept. To become tractable, the key concept to be brought into play is that **mm** is, above all else, a number, albeit a synthetic number. Numbers ultimately regulate quantity; this applies even to synthetic numbers. Synthetic numbers must regulate quantity *qualitatively*. Synthetic numbers are different from their analytic siblings in that they are universals. A universal number thus cannot ever be seen to be greater or less in magnitude than any other. They are order free. Given the order free universality requirement, somehow we must solve equation (17). This might seem like a task for an alchemist rather than a mathematician, but there must be a way. This is not a mathematical problem but a problem in anti-mathematics.

We proceed to find a solution to (17) as follows. The **mm** dyad can be considered as infinitesimally small – the smallest measure possible in the system or organism in which it participates. We will denote it by the evocative symbol  $\epsilon$ , which just makes the dyad easier to imagine. Thus **mm** =  $\epsilon$  where  $\epsilon$  denotes a dyad of infinitesimal extent and undetermined orientation. As such, it comports like a real valued scalar magnitude. Substituting in (17) gives

$$\epsilon = \epsilon^2 + 1 \tag{18}$$

the square  $\epsilon^2$  will be effectively zero and so, essentially  $\epsilon = 1$  whilst at the same time  $\epsilon^2 = 0$ .

Here, we see the primordial role played by the *mm* dyad in the system. Since  $\varepsilon = 1$ , the dyad acts as the unit of measure. All constructs will involve multiples of  $\varepsilon$ . At the same time, like the *ff* dyad, the *mm* dyad squares to zero. In addition, the inner product of the *mm* dyad with any other dyad, including itself, will be zero. The *mm* dyad is orthogonal to all other dyads present. It is the *singular* dyad par excellence.

Interpreting (18) this way may be thought of as pseudo-mathematics, which it probably is. However, it does illustrate the concepts in play. A more geometrical approach to understanding the double masculine will be given further on.

The *mm* dyad, in the present context of the first Stoic syllogism, can be thought of as a *real* entity, a real entity being that where all the essentials of the entity are present in the same moment. For the *mm* dyad both the first quality and the second quality are in possession. Both qualities are present *Now*. The essential qualities of the *mm* dyad are not present piecemeal like some sort of jerky movie consisting of a sequence of snapshots. That is the diachronic perspective of the traditional sciences. Instead, the real articulated by the *mm* dyad is synchronic. In the synchronic perspective, the organism or system is viewed as a whole, where all essentials are present in the same moment.

## 5 Cartesian and Non-Cartesian Geometry

There are two kinds of geometry, one analytic and one synthetic. Analytic geometry is traditional left side version and is based on analytic numbers. Our objective is to develop the right side version, synthetic geometry based on synthetic numbers.

The move from pure number to geometry involves formalising the notion of oriented number first introduced by Grassmann. In analytic geometry, the oriented number is interpreted as a vector. A vector space  $V^n$  is based on an ordered set of  $n$  orthonormal basis vectors  $\{e_1, e_2, \dots, e_n\}$ . The ordering of the set is arbitrary but does establish a necessary convention for distinguishing one axis from another. This is the basis for *Cartesian geometry*. The form of a geometric space depends on its metric and is defined by its signature  $(p, q, r)$ .

In the analytic approach, a vector space is not considered a thing: things live *in* spaces, not are spaces. This establishes a dichotomy between the space non-thing and the thing. This space-thing dichotomy is not tolerable in synthetic geometry because of the non-dualist demands of FC. The thing and the space it lives in must all be part and parcel of the one thing. Analytic geometry also violates FC with the Cartesian form of its basis vectors. As for the 3-dimensional space illustrated in Figure 6(a), the basis vectors all share a common point, the origin. This establishes the origin as a privileged point and so violates FC.

In synthetic geometry, there is no notion of a thing living in space, the thing and its space are synonymous, and so there is no FC non-dualism violation. FC also dictates that the basis elements must be synthetic numbers, not analytic. This means that the basis elements must come from the four-letter alphabet of synthetic numbers  $\{mf, ff, fm, mm\}$ . Moreover, the basis elements cannot be arranged in the Cartesian form. As discussed in SON, the only configuration of basis elements allowable is a triad of the form shown in Figure 6(b), or a multiple of such triads. In this configuration, there is no privileged point. These are called RGB triads and are self-labelling, another implicit requirement of FC.

As a passing comment, any dyadic, triadic or whatever kind of structure that violates the RGB convention should be considered not to consist of the same kind of matter as that making up an RGB based organism. Relative to the RGB based organism, non-RGB compatible matter would be treated as a kind of *antimatter*, a violation of FC, and a threat to the systemic non-dualist integrity of the organism. It is the responsibility of each organism to maintain its FC integrity. This is the driving force in the dynamics of such systems. The loss

of FC is tantamount to the loss of Self. In this paper, we concentrate on the static aspect of FC organised structures.

In synthetic geometry, the elementary geometric spaces all have the same RGB backbone and are thus 3-dimensional. Geometries that are more complex can be built up from these triads. The geometric form of these three dimensional entities is determined by the typing of the three dyads making up an RGB triad. Instead of being typed by the different kinds of analytic number, the dyads are typed by the four different types of synthetic number  $\{mf, ff, fm, mm\}$ . The first three of these synthetic numbers are imaginary and the fourth  $mm$  number is real. However, the extreme relativity of synthetic geometry imposed by FC demands that the real synthetic number can sometimes pose as an imaginary number and vice versa. A dyad in an RGB triad can thus be typed with any of the four binary genders.

Like any analytic geometry, the form of a synthetic geometry can be characterised by its signature. The traditional left side signature is defined by the triplet of numbers (p, q, r) denoting the number of basis elements with positive, negative and zero square. The right side version will be but a triplet of synthetic letters (r,g,b) where r is the binary typed Red basis element, g the binary type of the Green basis element, and b the binary type of the Blue. Thus, for example, generic geometry signature could be written as  $(ff, fm, fm)$ ,  $(mf, ff, fm)$  and so on. There are 64 possible signatures, a much richer structure than the analytic version.

SON posed the *Generic-Genetic Conjecture* that the binary gender coding  $\{mf, ff, fm, mm\}$  matched up with the RNA coding of the genetic code  $\{a, u, g, c\}$ . Until ever proven wrong, we will adopt this single letter encoding of the four elementary binary typed synthetic numbers. Using this format, for example, the signatures  $(ff, fm, fm)$ ,  $(mf, ff, fm)$  can be simply written as  $(ugg)$  and  $(aug)$  respectively.

Using RNA lettered signatures we will later show in more detail than in SON how a classical 2-dimensional Minkowski space  $R(1,1)$  of spacetime geometry has a *right* side version  $G(aug)$ . In everyday language, we intend to show how the two dimensional Minkowski space of spacetime geometry is related to the start codon AUG of the genetic code.

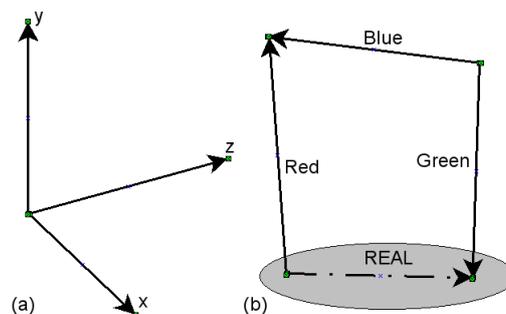


Figure 6 (a) Left side geometry is based on Cartesian axes. (b) The three bases for right side geometry are self labelling and form the “imaginary” attributes of a “real” entity. Unlike in Cartesian geometry, it may not be logically possible that the three morphisms of the triad be orthogonal to each other.

From a right side, perspective, left side geometry will appear very nobbled: Like the left hemisphere of the biological brain, left side mathematics seems to suffer from a form of hemineglect. Psychologists have observed that stroke victims with a compromised right hemisphere display bizarre bi-lateral oriented behaviour. The left hemisphere owns the right side world and so the patient might only eat food on the right side of the plate, only shave the right side of the face, only wash the right side of the body and so on. The left hemisphere only is conscious of only a half world. This is not the case for the right hemisphere. Despite being incapable of stringing words of a sentence together, the right hemisphere is conscious of a *whole* world. The indications are that the right hemisphere lives in the synchronic *Now*,

is weak on syntax, strong on semantics. Left side mathematics displays its own kind of hemineglect. Traditional mathematics is only conscious of *i* type and *j* type imaginary numbers, that is to say *mf* and *fm* typed mathematical entities. These entities square to non-zero values and hence appear as true quantities. All left side mathematics and accompanying sciences, because of analytic number foundations, are totally at sea when confronted with geometric entities devoid of determined quantity like the *ff* and *mm* typed mathematical entities that square to zero for example. Left side geometry only deals with a limited kind of geometry. The geometry is totally oblivious to the utility of the “degenerate” singular lines and does not treat the lightlike lines as of equal status to the timelike and spacelike lines. As such, and in many other cases, the left side take on geometry displays an inbuilt inability to discern the finer semantics of geometric forms.

Our next step towards the Universal Operational Calculus based on *now numbers* is to examine a crucial stepping-stone. The stepping-stone is called Geometric Algebra and its secret weapon is the geometric product arising from the work of Grassmann, Hamilton and Clifford in the nineteenth century.

## 6 Geometric Algebra

### 6.1 The Agenda

The road towards Geometric Algebra starts with Leibniz with his idea of *Analysis Situs*, a generalized geometry of situation not involving numbers. The algebra should be simple and simplifying and free the intellect from the “burden of imagination.” The new science should be capable of explaining the form of things including “plants and animals” and do so using only a few letters of an alphabet. It is realising this vision of Leibniz that provides the objective of this work.

For the author, Leibniz’s wish to remove the “burden of imagination” can be interpreted as eliminating any dependence on abstraction and moving to a non-diachronic, operational approach, free of abstract reasoning and inference. The science must be right side, not left side and thus constructed from generic universals rather than the generalised abstractions that form the basis of all present day sciences and mathematics. The few letters of the alphabet become the four letters of the generic-cum-genetic code. The freedom from numbers will be accomplished by replacing the role of analytic numbers, the quantitative, with synthetic “*Now*” numbers, the carriers of the generic qualitative. The elementary *now numbers* are none other than the four letters of the Code. According to the author, any system driven by the demands of FC will be explainable in terms of its aggregate *now number*, by its genome, so to speak. In this way, the whole thing is a *now number*. That, briefly, is the agenda.

#### 6.1.1 The Strategy

Inspired by Leibniz, Grassmann, as well as Hamilton and Clifford, pioneered an algebraic approach to geometry that eventually became known as Geometric Algebra (GA). However, GA is not geometry without number. Nevertheless, unlike traditional geometry, it is without coordinates.

Over recent times there has been a revival of interest in GA. David Hestenes has advanced the GA approach to provide new insight into a wide range of physical topics from classical mechanics and electromagnetism, to Quantum Mechanics and *gauge* theory. He has even shown how GA can be applied to geometries without metrical properties. Hestenes and others claim that GA is the universal algebra that unifies mathematics and physics (Hestenes, *Universal Geometric Algebra* 1988). Certainly, the resulting geometry is simple and simplifying. In the form of Conformal Geometric Algebra, its prowess has been exploited in

computer graphics. The complex graphics of any modern video game, more often than not, would probably have been developed on a GA platform. However, GA in its present form does not match up to the lofty vision of Leibniz.

In order for algebra of geometrical form be truly universal, it should have universal foundations. Instead, GA has the same analytic basis as traditional Hilbert style geometry. This paper proposes an alternative formulation based on a synthetic foundation using *now numbers*.

### 6.1.2 The Tool - the Geometric Product

Left side geometries are based on an abstract n-dimensional vector space defined over an abstract algebraic field such as the real numbers, the complex numbers, or the quaternions. It is the specificity of the field chosen that determines the metric properties of each space so defined.

Conventional expositions of GA also start from an n-dimensional vector space and hence have acquired a distinct left side slant right from the start. However, from a right side perspective the redeeming feature of GA is its unifying concept of the geometric product coupled with the mantra “Everything is a geometric entity” – a mantra much like the OO mantra “Everything is an Object.” In this respect, GA can be said not to violate FC. The geometries of GA are non-dualist in this respect.

In traditional left side geometries manipulation and modification of geometric objects is handled by matrices and tensors. GA must respect its mantra and the only possible modification of geometric entities is with other geometric entities. The mechanism used is the *geometric product* of one geometric entity with another. In the context of GA, the verb acts in the same way as the noun. For example, rotations or rotors are geometric entities. The quaternions discovered by Hamilton are examples of rotors. Armed with this kind of geometric entity the rotation of a geometric form can be described by the geometric product of the form with a rotor entity.

In brief, GA is a strong proponent of FC. There is no absolute dichotomy between entities and actions on or by entities. Even the act of sitting on a chair is a geometric entity according to this mantra. It is worthwhile noting that the GA mantra resembles the mantra of the Stoics notably that “Everything that exists is a corporeal body.” Corporeal bodies must have extent and be capable of acting on or being acted on.

The algebra of GA is based on the geometric product  $\mathbf{ab}$  of two geometric entities  $\mathbf{a}$  and  $\mathbf{b}$  which presumably articulates the fact of  $\mathbf{a}$  acting on  $\mathbf{b}$ . However, what about the situation  $\mathbf{ba}$  of  $\mathbf{b}$  acting on  $\mathbf{a}$ ? What we have here is the beginning of the dialectics of the active and the passive, another favourite topic of the Stoics. In general,  $\mathbf{a}$  acting on  $\mathbf{b}$  is not the same as  $\mathbf{b}$  acting on  $\mathbf{a}$ , and so the geometric product is anti-commutative. GA exploits the fundamental asymmetry between the active and the passive to construct the geometric product. The geometric product  $\mathbf{ab}$  is defined as the sum of two other products, the inner product  $\mathbf{a}\cdot\mathbf{b}$  and the outer or wedge product  $\mathbf{a}\wedge\mathbf{b}$  due to Grassmann.

The geometric entities  $\mathbf{a}$  and  $\mathbf{b}$  are composites of oriented numbers. GA assumes that the numbers are analytic. Our intention is to indicate how the approach could work for a *synthetic* number version.

The geometric product can be decomposed into two other products, one symmetrical and one anti-symmetrical

$$\mathbf{ab} = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\wedge\mathbf{b} \tag{19}$$

The symmetrical part is given by the inner product

$$\mathbf{a}\cdot\mathbf{b} = \frac{1}{2}(\mathbf{ab} + \mathbf{ba}) \tag{20}$$

The non-symmetrical part is given by the outer product

$$a^b = \frac{1}{2}(ab + ba) \quad (21)$$

The symmetric and anti-symmetric properties of the two products are expressed by :

$$a.b = b.a \text{ and } a^b = -b^a \quad (22)$$

In brief, the principle requirement for a non-trivial (geometric) product is that the relationship between the passive and the active entities be non-symmetrical.

### 6.1.3 Generic Basis for GA

Left side mathematical science is strong in its abstract generalisations: everything is a special case of this or a special case of that. In order to accomplish abstraction, conventional mathematics needs higher order logic where variables can range over domains such as sets or sets of sets and so forth. The price paid for such expressive power is its weakness in dealing with form. From the left side perspective, form becomes the simple property of an entity. Characterisation of the property inevitably breaks down into an ensemble of scalar attributes.

A good example is Particle Physics where the elementary particles like quarks, leptons, and bosons are said to be totally devoid of form. They are seen as point-like entities with no internal structure whatsoever. These point-like entities flit around in a void and somehow manage to carry around in their non-existent interiors a small baggage of scalar attributes such as electric charge, rest mass, spin and a few others. All matter is supposed to be composites of these zero sized particles, all floating around in empty space. An elementary calculation leads to the absurd result that the total percentage of space occupied by matter in the universe is zero. This conclusion may be absurd but is not wrong. It is probably the correct conclusion made from within the confines of the left side a priorist paradigm. The picture given is not the way the world *is*, but the way it inevitably appears when viewed through the prism of a particular paradigm. In the end, it becomes more a reflection on the prism than the world.

When it comes to geometry, left side mathematics makes a simple generalisation of scalar properties leading to the left side version of a geometric property. Geometry becomes nothing more than a higher dimensional generalisation of the scalar property. The scalar is replaced by the n-tuple of scalars. The scalar itself can be generalised from a real valued number to that of the complex valued number – an ordered doublet of scalars, or perhaps to a quaternion as a four-tuple of scalars. A popular geometric space for physical applications is an n-dimensional Hilbert space defined over a field of complex numbers – a space made up of n-tuples of doublets of scalars. Left side geometry is very scalar oriented. According to modern physics, lots of particles “live in” this kind of space. This is what we mean when we say that, although left side science deals with high order logic, it only can handle zero order semantics, the semantics of the scalar valued attribute, the quantitative measure, the analytic number.

We characterise right side science as being based on zero order logic thus variables are disallowed to range over a domain and hence abstraction becomes impossible. The strength of the right side paradigm is that it allows higher order semantics. This means that the scalar oriented “attribute based” approach of left side science must give way to geometric representations. The first thing to go is the notion of real and imaginary valued scalars that make up complex numbers. The complex number must give way to a geometric representation. In this respect, even traditional GA shows the way. As some GA proponents have commented:

... complex numbers arising in physical applications usually have a natural geometric interpretation that is hidden in conventional formulations. (Gull, Lasenby and Doran 1993)

GA aspires to replace the scalar oriented approach with a focus on geometric form. GA starts with the same abstract basis  $E$  of orthonormal basis vectors  $\{e_1, e_2, \dots e_n\}$  as for left

side mathematics. Despite this similarity, GA avoids treating the metric properties of the basis as scalars and in particular as complex numbers. Instead, the properties of the basis vectors are determined geometrically using the geometric product. GA declares that there are three possibilities defined by the geometric product of the basis with itself. Since  $e^2 = e \cdot e + e \wedge e$  and  $e \wedge e = 0$  then  $e^2$  will be a scalar simply given by  $e^2 = e \cdot e$ . The three possibilities are:

$$e^2 = 1, \quad e^2 = -1, \quad \text{or} \quad e^2 = 0. \quad (23)$$

Thus, in GA the signature  $(p,q,r)$  of a space  $G$  is defined by the number of abstract basis elements  $p$  that square to 1,  $q$  that square to -1, and  $r$  that square to 0. The signature is thus determined by the geometric product of geometric entities and not by the properties of elements of an algebraic field, as in the conventional case.

Overall, GA makes a good fist at emphasizing geometric form rather than taking the usual attribute based, analytic, scalar oriented approach of abstract, matrix and tensor dominated conventional geometry. However, is this a good thing? Is GA better? The proponents of GA provide many reasons why a GA approach to real world physical applications is preferable to conventional methods. Many, like Gull et al, express their frustration, noting that the pioneering work of David Hestenes over the years has been poorly understood and the pressing need that “his message be AMPLIFIED and stated in a language that ordinary physicists understand” concluding that “the geometric algebra of spacetime is the best available mathematical tool for theoretical physics, classical or quantum. Practitioners of Computer Graphics also promote the power, simplicity, and computational advantages of the Conformal Geometric Algebra refined and popularised by Hestenes. (Dorst, Fontijne and Mann, Geometric Algebra for Computer Science, An Object Oriented Approach to Geometry 2007)

We employ a different strategy in order to promote the approach. Our strategy is two pronged: first provide a more fundamental distinction between conventional geometry and GA and secondly, propose using a radically different kind of basis for geometry than an  $n$ -tuple bunch of analytic numbers. Thus, we must first of all indicate in what fundamental way GA is different from the conventional approach. One way of looking at it is to interpret GA as an embryonic *operational* form of geometry. Understanding that GA has *operational* credentials helps in situating the discipline.

However, it should be understood that both the operational approach to analysis, as well as the GA approach to geometry, are still fundamentally left side sciences as they totally rely on analytic numbers. They may very well introduce universal methodology into the domain of ordinary abstract mathematics, but the ground has already been tainted. The same also applies to Category Theory, as it too attempts to provide universal constructs within an already generalised, abstract, and axiomatised world. The universal, in order to be truly universal, must be free of such *a priori* constructs.

This is where conventional GA comes up against a formidable obstacle. Hestenes claims that GA provides the universal algebra and geometry of mathematical physics. The obstacle is that GA is not truly a universal science. Rather, it is an attempt at a universal science that retains much the same foundations as classical abstract geometry. Traditional abstract geometries are founded on an  $n$ -tuple of abstract basis elements with signature  $(p,q,r)$ . As Gull et al point out in their paper, the treatment and interpretation of the abstract basis is different in GA. Nevertheless, the basis of the geometries in both cases is abstract and analytic. The elements of the abstract basis have no meaning except for how they square, a quantitative measure. The abstract basis only has zero order semantics – each basis element squares to 1, -1, or zero;; end of the semantics story.

It is here that we come to the most radical and most important innovation reported in this work. Instead of founding our universal geometric algebra of Nature on an abstract basis, we propose founding it on the most universal basis possible. We propose the totally generic and

non-abstract basis composed of only two entities – the generic masculine entity  $m$  and the generic feminine entity  $f$ . The basis for our Universal Geometry Algebra (UGA) becomes  $\{m, f\}$ . As for the semantics, the feminine corresponds to pure content without form, pure Manyness, the masculine corresponds to pure form without content, pure Oneness.

These two generic entities  $m$  and  $f$  correspond to the two most primordial geometric constructs of universal geometry, the construct arising from the draconian demands of constructing form and content that does not violate FC.

#### 6.1.4 The Generic Basis Generates the Four Generic Bases

Conventional GA apes abstract geometry by assuming an initial basis for the geometry is a set  $E$  of orthonormal basis vectors  $\{e_1, e_2, \dots, e_n\}$  of an abstract vector space  $V^n$ . We will call the basis set  $E$  of the vector space  $V^n$ , the *abstract basis*. GA does not use the abstract basis  $E$  as its basis set. Instead, it constructs what is called the *standard basis* by taking all the geometric products of the basis elements in  $E$ . The author's preferred terminology is that the standard basis be referred to as the *bases* of the geometry. In the case for  $n=3$  the abstract basis will be of the form  $E = \{e_1, e_2, e_3\}$  and calculating all the geometric products between elements gives a set of bases  $E'$  where:

$$E' = \{1, e_1, e_2, e_3, e_1e_2, e_1e_3, e_2e_3, e_1e_2e_3\} \quad (24)$$

Thus, for an abstract basis of 3 elements, there are 8 bases. In general, an underlying  $n$  dimensional vector space produces a  $2^n$  dimensional GA.

In the case where  $n=2$ , the GA geometry will have the bases  $E = \{e_1, e_2\}$  where  $e_1$  and  $e_2$  are two orthonormal vectors. In GA the basis  $E$ , together with the geometric product is used to generate the bases for a Clifford Algebra with the four bases  $E' = \{\sigma_0, \sigma_1, \sigma_2, \sigma_3\}$ , where

- $\sigma_0$  is the unit scalar or zero-vector  $1$  generated by  $e_1^2 = 1$  and  $e_2^2 = -1$
- $\sigma_1$  and  $\sigma_2$  are equal to the one-vectors  $e_1$  and  $e_2$ .
- $\sigma_3$  is a bi-vector pseudoscalar equal to the geometric product of  $e_1$  and  $e_2$ . Since the dot product  $e_1 \cdot e_2$  is zero,  $\sigma_3 = e_1 \wedge e_2$ .

In general, geometric objects in the geometric space will be multi-vectors made up of a linear combination of  $m$ -vectors where  $m < 2^n$ . A multi-vector  $v$  for  $n=2$  will be of the form

$$v = a\sigma_0 + b\sigma_1 + c\sigma_2 + d\sigma_3 \quad (25)$$

In the case of the Clifford Algebra  $C(1,1)$ , the base  $e_1$  will square to  $+1$  and  $e_2$  will square to  $-1$ . Of particular concern in moving to our objective of a truly synthetic geometry is the ordinal nature of the bases. The four bases of  $C(1,1)$  can even be ordered by their dimensionality or grade.. Only the bases  $e_1$  and  $e_2$  are of the same grade. The base  $\sigma_0$  is of grade zero and  $\sigma_3$  is of grade 2. Quite clearly, this situation is a violation of FC and so is unacceptable. Even from an aesthetic point of view, the author has always found that GA constructs with scalars and pseudo-scalars always seems to have a clunky air about them.

In order not to violate FC we propose replacing the traditional Cartesian basis  $E = \{e_1, e_2\}$  with a generic pair of *now numbers* to make the generic basis  $B = \{f, m\}$ . This leads to a generic version of the Clifford Algebras with a generic set of bases generated by the geometric product. The four bases *Now* become  $B' = \{mf, ff, fm, mm\}$ . Using the conjectured RNA encoding version  $\{mf, ff, fm, mm\} = \{a, u, g, c\}$ .

#### 6.1.5 From Analytic to Synthetic Signatures

In classical GA, the Clifford Algebra  $C(p,q,r)$  generated from the  $n$ -dimensional vector spaces  $R(p,q,r)$  will have  $2n$  base multi-vectors. Introducing our right side alternative to GA, the basis made up of the two elementary  $f$  and  $m$  generates an alphabet of four bases  $B' = \{a, u$

**g cj**. Any triad of these bases defines an elementary spatial unit  $G(x,y,z)$  that we called a *codon*, where  $(x,y,z)$  is the signature of the spatial unit made up of letters chosen from the four letter alphabet  $B'$ . Borrowing from Leibniz, we will sometimes refer to a codon as a *situation codon* or just simply a *situation*. What we are trying to accomplish is to develop Leibniz's *Analysis Situs*, not as Poincare's topology but as a generic geometric algebra based on synthetic number. Leibniz might also have used the term *monads* for these elementary situation codons when viewed in a wider perspective. In what follows, we will look at the elementary "situations"  $G(\mathbf{aug})$ ,  $G(\mathbf{ggg})$  and  $G(\mathbf{ugg})$ .

## 6.2 The Geometric Algebra of the aug codon

The four generic bases **a**, **u**, **g**, and **c**, are gendered *mf*, *ff*, *fm*, and *mm*, respectively. Each base can be represented by an oriented dyad. These four types of dyad are the right side version of vectors in analytic geometry. All analytic geometries share the same identical affine structure. The only typing of vectors is by their metric properties. Vectors all share a common fixed point, the origin. The other end of a vector is also fixed, being determined by its coordinates. Unlike vectors, the ends of dyads are not all predetermined locations; they may have degrees of freedom. Binary gender determines the degrees of freedom of each end of the dyad. If the dyad end is typed masculine, it is has zero degrees of freedom. A feminine typed end is unrestrained and so corresponds to non-zero degrees of freedom. The non-zero value is indeterminate except that it is non-zero. Thus, we can simply talk about a masculine typed end being fixed and a feminine typed end being free. Note that fixity and freedom is not an absolute concept, but a relativist determination. What appears as fixed in one situation may be free in another.

Both the **a** type and the **g** type dyads have one fixed end and one free but have opposite orientations. Both ends of the **u** type dyad are feminine and thus both are free. By contrast, a **c** type dyad has both ends masculine and hence fixed.

In passing, note that vectors, if interpreted as dyads, would have both ends masculine typed. The vector would thus appear as a **c** typed dyad. The inability to deal with the feminine is a common characteristic of all left side sciences and mathematics; all entities are assumed neuter gendered where the masculine plays the neuter role.

We *Now* consider a simple jigsaw puzzle of connecting **a**, **u**, **g**, and **c**, typed dyads together in a self-constraining structure so that the freedoms of the perfectly free feminine ends of dyads somehow cancel themselves out. Thus, masculine ends get connected to masculine ends and feminine to feminine. The FC principle demands that the structure must be an RGB triad determining the "imaginary" part of the structure, the ends of which determine the real. The real part will be a **c** typed dyad. The necessary fit matches the two free ends of the **a** and **g** dyads with the two free ends of the **u** dyad to form the **aug** structure shown in Figure 7. The **aug** situation codon determined in this way literally plays the role of "start codon" in the science of *Now*.

The source end of the **c** and **a** dyads correspond to any-point-whatsoever from an individual perspective. This point could be thought of as "somewhere," the subjective starting point. The head of the **c** and **g** dyads corresponds to the objective, generic starting point, the real "centre of the universe," so to speak. The subjective, individual starting point and the generic starting point form the two ends of the **c** dyad and so are different. If it is assumed that the **c** dyad has zero norm, the distance between the two ends is effectively zero and so the two points are indistinguishable. The **aug** codon represents the situation where the *Now* starts – anywhere whatsoever.

Viewed simplistically, the geometry of gender typed dyads can be imagined as hinged structures. Some dyads can also be considered "rubbery" so that the structures can change

dimension as well as direction. In their ensemble, the hinged, rubbery structures cover an immense range of possibilities only constrained by the non-violation of FC. The **aug** situation codon is one of the simplest and most fundamental of these hinged, rubbery geometric forms.

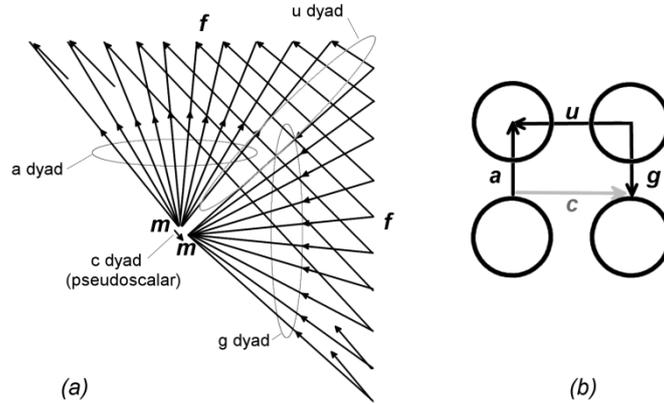


Figure 7 (a) Illustrating the natural fit of the **a**, **u**, and **g** dyads to determine the "real" **mm** typed **c** dyad. (b) The Chrysippus diagram for the **aug** codon.

### 6.2.1 Mathematics of the **aug** codon

The proper synthetic mathematics based on *now numbers* does not yet exist. Thus, the author is obliged to take a leaf from Heaviside and engage in some "mathematical witchcraft" by using a mishmash of analytic and synthetic methods. To start with, we will ignore the **mm** typed **c** dyad illustrated in Figure 7. We will assume that the **c** dyad is null and so its two endpoints will represent a single point at the origin. Thus, the **aug** codon structure becomes planar and can be interpreted as a two-dimensional Minkowski space. It is fruitful to explore the Minkowski plane structure using ordinary Geometric Algebra. Adopting Hestenes notation and following the theme in his paper,  $R^{p,q}$  will denote a vector space with signature  $(p,q)$ .  $R_{p,q} = G(R^{p,q})$  denotes the geometric algebra generated by  $R^{p,q}$  (Hestenes, *Old Wine in New Bottles: A new algebraic framework for computational geometry* 2001).

Consider the Minkowski plane  $R^{1,1}$  with the orthonormal basis  $\{e_+, e_-\}$  where

$$e_+^2 = 1, \quad e_-^2 = -1, \quad \text{and the dot product } e_+ \cdot e_- = 0 \quad (26)$$

$R^{1,1}$  generates the geometric algebra  $R_{1,1}$  with basis  $\{1, e_+, e_-, E\}$  where  $E$  is the pseudoscalar defined by

$$E = e_+ \wedge e_- \quad \text{and has the property } E^2 = 1 \quad (27)$$

Without loss of generality the basis of  $R_{1,1}$  can be replaced by the basis  $\{1, n_+, n_-, E\}$  where

$$n_+ = e_+ \exp(i\phi) \quad \text{and} \quad n_- = e_- \exp(-i\phi) \quad (28)$$

and once again

$$n_+^2 = 1, \quad n_-^2 = -1, \quad \text{and the dot product } n_+ \cdot n_- = 0 \quad (29)$$

which illustrates that the two bases  $n_+$  and  $n_-$ , although of determined magnitude, *have indeterminate angle*. Thus these two dyads have the same semantics as the **a** and **g** dyads and are the conjugate of each other.

Now reconsider the same Minkowski plane  $R^{1,1}$  with a different basis, the null basis  $\{e, e_*\}$ . The basis elements in this case are lightlike and, in the context of the **aug** structure, will be related to the orthonormal basis elements by

$$e = (e_+ + e_-) / \sqrt{2} \quad \text{and} \quad e_* = (e_+ - e_-) / \sqrt{2} \quad (30)$$

which gives the null basis properties:

$$e^2 = e_*^2 = 0 \quad \text{and the dot product } e \cdot e_* = 1. \quad (31)$$

In this case, the basis elements have determined direction but they *have undetermined magnitude*. Once again, without loss of generality, we can replace the null basis  $\{e, e_*\}$  with the null basis  $\{n, n_*\}$  where

$$n = \lambda e, \quad n_* = e/\lambda, \quad n^2 = n_*^2 = 0, \quad \text{and} \quad n \cdot n_* = 1 \quad (32)$$

which generates the geometric algebra basis  $\{1, n, n_*, E\}$  with the unit pseudoscalar  $E = n \wedge n_*$ .

This null basis  $\{n, n_*\}$  implements the semantics as the  $\mathbf{u}$  dyad with the freedom of both its endpoints expressed as undetermined magnitude. On the orthonormal basis side the pair  $\{n_+, n\}$  implements the semantics of the  $\mathbf{a}$  and  $\mathbf{g}$  dyads. As for the semantics of the  $\mathbf{c}$  codon in this *aug* configuration, that is more complex.

In the traditional left side treatment of spacetime geometry there is never any mention of the  $\mathbf{c}$  codon. The only geometric elements are the cone of timelike lines, the cone of spacelike lines and the lightlike lines. These correspond to the  $\mathbf{a}$ ,  $\mathbf{g}$  and  $\mathbf{u}$  dyad elements. What traditional spacetime geometry ignores is the real *raison d'être* of the *aug* geometry. Instead of understanding it in terms of the Special Theory of Relativity of physics, the construct can be interpreted from a much more universal and generic perspective. Standard spacetime diagrams only show the projection of the *aug* construct onto the individual Minkowski plane. Ignored is what lies *outside* of the individual plane  $R^{1,1}$  – the generic end points of the  $\mathbf{c}$  dyad. Informally including the generic  $\mathbf{c}$  dyad (as vector) into the geometry, increases the dimension by one and provides a representation space where, parallel to the Minkowski plane  $R^{1,1}$  lies another plane which we will refer to as the *generic* plane P. The representation space will be denoted by  $R^{\text{aug}}$  and is given by the direct sum  $R^{\text{aug}} = R^{1,1} \oplus P$ . Any vector  $\mathbf{X}$  in P will have the form  $\mathbf{X} = \mathbf{x} + \mathbf{c}$  where  $\mathbf{x}$  is in  $R^{1,1}$ .  $\mathbf{X}$  is the  $R^{\text{aug}}$  representation of any  $\mathbf{x}$  in the projection plane  $R^{1,1}$ . The algebraic properties of *care*:

$$\mathbf{c}^2 = 0 \quad \text{and} \quad \mathbf{c} \cdot \mathbf{n} = \mathbf{c} \cdot \mathbf{n}_* = \mathbf{c} \cdot \mathbf{n}_+ = \mathbf{c} \cdot \mathbf{n} = 0 \quad (33)$$

Thus,  $\mathbf{c}$  is orthogonal to all lines present including itself. Such lines are not timelike, nor lightlike, nor spacelike; but like what Goldblatt calls *singular lines* (Goldblatt 1987).

### 6.2.2 The *raison d'être* of the $\mathbf{c}$ dyad

Because of the apparently degenerate nature of singular lines, the projective geometry of  $R^{\text{aug}}$  may seem to border on the profoundly degenerate. However, the geometry hides some interesting nuances when considered from the point of view of infinitesimals.

In analytic mathematics, the formalisation of infinitesimals has already been solved in the form of Non-Standard Analysis. The numbers of Non-Standard Analysis are a superset of the real numbers that includes infinitesimals that can be treated as numbers in their own right. This is an interesting example of left side mathematics becoming more FC. However, Non-Standard Analysis will still violate FC on other counts, such as its axiomatic beginnings. Non-Standard Analysis only provides an *abstract* version of the infinitesimal and, as such, has nothing much to do with actual reality. Our judgment here assumes that the prime normative arbitrator of actual reality is the principle of FC and not something else like axioms, measurements, scientific hunches, opinions and so on.

Thus, what the generic science of synthetic *now numbers* requires is a generic non-abstract version of the infinitesimal. In the context of the *aug* codon construct, it is clearly the  $\mathbf{c}$  dyad that plays the role of infinitesimal. With the  $\mathbf{c}$  dyad assumed null, the AUG construct becomes a closed triad with a distinguished point – the null  $\mathbf{c}$  dyad. The space is not homogeneous. With a non-null  $\mathbf{c}$  dyad, the space is extended by augmentation with the generic plane P to produce the homogeneous space  $R^{\text{aug}} = R^{1,1} \oplus P$ . In the homogeneous space  $R^{\text{aug}}$ , the source of the  $\mathbf{c}$  dyad represents the empirical origin, whilst the target will be a point

in P and so represents the generic origin  $g$ , common to all *aug* codons. Intuitively using traditional analytic mathematics

$$g = \varepsilon c \quad (34)$$

where  $\varepsilon$  is infinitesimally small, whatever that may mean in generic terms.

In addition to the origin there is also be the generic “point at infinity,” denoted by  $g^*$ , which is lacking from the projection plane and must also be in the generic plane P. Thus

$$g^* = c/\varepsilon \quad (35)$$

### 6.2.3 The Generic Infinitesimal

It is *Now* time to change gears, stop thinking in analytic terms, and move to a generic approach. Just as the infinitesimal is a non-trivial, rather enigmatic entity in analytic mathematics, the infinitesimal in synthetic mathematics is even more enigmatic. Very briefly, we describe the unifying but surprising aspects of the generic infinitesimal  $\varepsilon$ . First of all, the number  $\varepsilon$  must be a *now number* and so cannot be greater or lesser magnitude than any other number. Consequently, from the above

$$\varepsilon = 1/\varepsilon = 1 = g^* = g = c \quad (36)$$

In this context, the generic infinitesimal is not an analytic number with magnitude, but a geometric entity, a synthetic number intrinsically embroiled with the generic plane P. We will not endeavour to pursue the intricacies of this generic infinitesimal unit here, except to do some hasty normalisation.

Unlike the  $a$ ,  $u$ , and  $g$  bases, the  $c$  base has zero degrees of freedom at both ends. Figure 7 illustrates how the degrees of freedom of the  $a$ ,  $u$ , and  $g$  bind together to determine the “fixed at both ends”  $c$  dyad. The fixed nature of the  $c$  dyad is expressed by the fact that it is the standard qualitative measure of the system. The  $c$  dyad is the generic unit and the generic infinitesimal, and much more.

Considered analytically, the  $a$  and  $g$  bases have undetermined angles  $\emptyset$  and  $-\emptyset$  respectively. The  $u$  dyad has two degrees of freedom in terms of magnitude, one along the  $n$  vector given by the undetermined value of  $\lambda$  and the other degree of freedom along  $n_*$  given by  $1/\lambda$ . The undetermined parameters  $\emptyset$  and  $\lambda$  are not independent. Simple trigonometry relates them by:

$$\tan(\emptyset) = \lambda^2 \quad (37)$$

If  $\emptyset$  was small, hence close to an analytic infinitesimal

$$\tan(\emptyset) \approx \emptyset \approx \lambda^2 \quad (38)$$

In a desperate attempt to wrap up with an operational approach, we definitively lapse into pseudo-mathematics. Very bravely, we eliminate all arbitrariness by substituting  $\lambda = m$  and so

$$\emptyset = \lambda^2 = m^2 = mm = c = \varepsilon \quad (39)$$

The idea is to make everything depend on the one single *Now*-number  $c$ . In so doing, the generic geometric entity space  $R^{aug}$  becomes normalised as a purely generic entity free of the arbitrariness of analytic numbers. The result will be independent of scale. The mathematics of synthetic number presented here will be found severely wanting at this stage, but this is one way at least to illustrate the intent of a science based on the generic.

All in all, there seem to be two kinds of solution for the generic infinitesimal. One is based on denoting the very smallest number  $\varepsilon$  and  $1/\varepsilon$  denoting the very largest number in the system. The other is where these two numbers are normalised to be equal, which is more difficult to understand but may have pertinence in some contexts.

Also, note that the only other uniquely coding codon like *aug* is *ugg*, which codes the amino acid Tryptophan. Interpreted from a spacetime geometry point of view, the space

would have no timelike lines. The nearest correspondence would be a manifold in Minkowski space that did not privilege any particular direction. This naturally leads to a de Sitter space interpretation. A de Sitter space is maximally symmetric and completely isotropic and thus, from our point of view, naturally satisfying FC. It can be thought of as a sphere in Minkowski space, the space of the Special Theory. Considered as a space in its own right, de Sitter space can be interpreted as the zero energy solution of Einstein's equations for General Relativity. We don't want to get caught up with the details here, but suffice to say that it has something to do with Dark Energy and the expanding universe. What interests us is purely the correspondence with the implicit generic geometry of the *ugg* codon. It is quite astounding that the two uniquely coding codons, *aug* and *ugg*, each correspond to a pillar of Relativity Theory in physics. The *aug* codon implements the Special Theory and the *ugg* codon, the General Theory. However, this is not a physicalist result, but a *generic* principle which apparently is expressed equally forcibly in the biological world.

### 6.3 The Degeneracy of the Genetic Code

There have been many attempts to apply mathematic to the genetic code, various kinds of parameterisation, grey codes, and even p-adic numbers, just to name a few approaches. Many of these attempts also endeavour to explain why the 64 possible codon combinations only code 20 amino acids and three stop codons – the so-called degeneracy of the genetic code. Our approach is not to apply mathematics to the genetic code but rather to consider the genetic code as a *natural* form of mathematics occurring throughout Nature. This mathematical code did not evolve but rather plays the role of the universal *a priori* in order for evolution of any kind to be at all possible.

Any kind of mathematics requires a normative mechanism in order that rational judgments are possible. Axiomatic mathematics relies on an *a priori* set of axioms for this purpose. The generic mathematics of Nature cannot depend on any such predetermined *ad hoc* construct. Generic mathematics must rely on the one universal *a priori* that there cannot be a determined *a priori*. We have interpreted this principle as being a demand for FC, an apparent *carte blanche* for “anything goes.” In reality, the constraints of FC are quite draconian and dictate the form and semantics of the genetic-cum-generic code. The analytic approach to semantics is via high order abstract reasoning. By contrast, the generic approach avoids abstraction by replacing abstract logic with a new, generic form of geometry that is capable of expressing higher order semantics, something that analytic methodology cannot do. Such a geometry must be capable of addressing the reality of any organism in the immediacy of its present, its “*Now*.” This is accomplished by replacing ordinary synthetic number with numbers devoid of any possible ordering—*now numbers*. The subsequent generic geometry opens the window on a monist reality by articulating the generic form of the elementary situations. This is indeed Leibnitz's *Analysis Situs*, the geometric algebra of situations.

The elementary situations can be enumerated as 64 elementary signatures formed from triads of letters from the {a, u, g, c} alphabet. Above, we illustrated how the geometric form G(aug) was compatible with FC, which can be interested from the point of view of Lorentzian invariance. The same can be accomplished with the form G(ugg) which can be shown to be the right side version of de Sitter space and hence an expression of General Relativity. In the genetic code, the codons aug and ugg are the only codons that uniquely code an amino acid or stop codon. All other combinations exhibit redundancy.

One example of redundancy is the ggg codon which codes the amino acid glycine. The corresponding generic geometric form would be G(ggg). Ignoring the sign of the metric, the traditional left side geometric space would be the 3-dimensional Euclidean space  $E^3$  in the form of  $R(0,3)$ .

In passing, it is interesting to note a few details concerning the biological expression of this generic form as the glycine molecule. For what it's worth, we note that glycine is the only protein-forming amino acid that is superimposable on its own mirror image; it is the only non-chiral amino acid. That looks a bit promising as there is nothing chiral about  $E^3$ . It is also the smallest of all the amino acid molecules. Thirty per cent of collagen is made up of this tiny glycine molecule.

What is interesting for our purposes is that  $ggg$  is not the only coding of glycine. The codons  $gga$ ,  $ggu$ , and  $ggc$ , are also encodings. This is an example of the so-called degeneracy of the genetic code and demands explanation. The only normative source of truth in Generic Science is that of FC. Thus, in some way, the  $G(ggg)$  generic geometric entity violates FC, but how? Fortunately, a ready-made geometric explanation has already been advanced by David Hestenes in his work on Conformal Geometric Algebra (CGA). Due to the achirality of the  $G(ggg)$  geometry the RGB triadic basis of  $G(ggg)$  leads to the same kind of geometry as the conventional Cartesian triplet  $\{e_1, e_2, e_3\}$  based  $R(0,3)$  geometry. The conventional simplistic Cartesian basis based geometries, are forcibly all achiral. Thus, in this case, the Hestenes' reasoning behind conventional CGA should carry over to the generic version.

CGA is an extension of a base space, an extension of  $R(0,3)$  in this case. The reason why this extension is necessary is that the origin for ordinary Euclidean space is a "distinguished point," as explained by Hestenes (Hestenes, *Old Wine in New Bottles: A new algebraic framework for computational geometry* 2001). This means that Euclidean space is non-homogenous. From our point of view, non-homogeneity is a violation of FC. At any rate, the Euclidean space construct is a violation of homogeneity. In order to attain homogeneity, the base space  $R(0,3)$  must be extended by two extra dimensions whilst *not increasing the number of degrees of freedom*. The role of the extra two dimensions was to provide two things that are missing from ordinary Euclidean space, notably a point at infinity and a generic origin that is no longer a distinguished point. The extended space can be implicitly expressed by the direct sum

$$R(1,4) = R(0,3) \oplus R(1,1) \tag{40}$$

where the generic origin and point at infinity are in the plane  $R(1,1)$ . (Hestenes prefers a proper GA approach based on direct product). Note that such an extension was not necessary for the  $G(aug)$  case as that geometry already effectively includes  $R(1,1)$ . The geometric space  $R(1,4)$  is a 5-dimensional Minkowski space. However, for the CFA approach, the direct sum must not increase the number of degrees of freedom, which must rest at three. This CGA constraint is achieved by reconsidering points in  $R(1,4)$  as vectors of zero norm. The projection of these vectors onto the base space  $R(0,3)$  correspond to the traditional notion of point in  $E^3$ . The end result is a homogenous space where rotations, translations, dilations, and reflections can all be treated in the same homogenous way. There are many universals like a circle being represented simply by a triple of points. A straight line becomes a special case of a circle of infinite diameter by one of the three points being the point at infinity. CFA is used extensively in computer graphics. Beauty abounds in CGA and is worth the time studying mathematically and practically via graphic engines (Doran, Lasenby and Lasenby 2002) (Dorst, Fontijne and Mann, *Geometric Algebra for Computer Science, An Object Oriented Approach to Geometry* 2007).

Figure 8 shows the Heraclitus diagram for the combined  $ggg$ ,  $gga$ ,  $ggy$ , and  $ggc$  codons that code the amino acid glycine. We argue that this is Nature's way of achieving homogeneity for the base geometry  $G(ggg)$ . Furthermore, we argue that the degeneracy of the genetic code can be explained in these terms – a consequence of achieving FC in the form of generic geometric homogeneity..

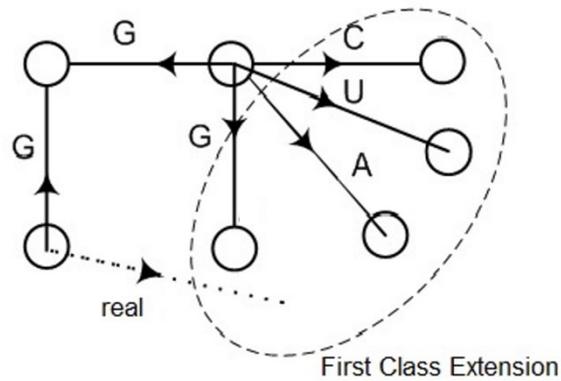


Figure 8 Nature's homogenous solution for the the first class extension of the ggg codon. This is the generic geometry version of the CFA construct based on  $R(1,4) = R(0,3) \oplus R(1,1)$ .

## 7 Conclusion

This paper, together with SON, can be looked at in two ways: through ancient eyes or through modern. Through the eyes of the ancients, we have shown how the one single principle of FC and the ontological gender construct, can unify the fragmentary remains of the ancients to reconstruct a unified non-abstract, generic kind of science. Our achievement in this regard has been to illustrate how the ontological flux of Heraclitus can be illustrated in what we have called Heraclitus diagrams. These same diagrams illustrate the underlying ontological structure of Aristotle's syllogistic logic and Square of Oppositions. The Heraclitus diagrams also illustrate the commonality of Stoic logic with the Syllogistic. However, it is with Stoic logic that we find the best fit for our purposes. Stoic logic is zero order like the propositional calculus. It is a logic of particulars and so free of abstraction. We claim that the Stoics used this kind of logic ontologically as a logic of substances. The Stoic often vaunted unity of Physics and Logic can be found here. In fact, we have illustrated how the Stoic syllogisms are directly related to the Empedocles theory of the four classical elements.

For the Stoics, the whole thing was held together by the principle of Virtue, the demands of a perfect world. We have modernised this concept and made it more tractable. Instead of Virtue, we speak about the principle of a world dominated by FC. Some things might display a disposition to violate FC, others a disposition to comply. Ultimately, for the organism conscious of what is good for itself, an exercised disposition towards the respect of FC has more than considerable merit. In this way, the Stoics' claim regarding the unity of physics, logic, and ethics can be realised and even become a tractable science – universally applicable.

The table in Figure 9 provides a summary of the universal nature of the approach. The fundamental underlying generic form is a Three-plus-One structure. Analytic sciences are always hell bent on pigeonholing things. Generic, synthetic science knows no such boundaries. The universality of generic structure can apply anywhere and at any scale and to anything that exists. For the rusted-on analytic thinker, this may be quite derailing and even threatening. However, unlike the stroke victim, the left side thinker has another half a brain to resort to and so, hopefully, can learn to eat food on both sides of the plate.

Viewed through modern eyes, we have illustrated where the action must be concentrated in order to progress. The progress of synthetic science requires a revolution in understanding semantics. The tool for such an understanding is a much more elaborate kind of geometry. The geometry will be along the lines of an algebraic geometry of situations postulated by Leibniz. The geometry will involve much more than points, lines, and planes. It will involve partially determined generic structures where only a half of the situation is ever determined. The fundamental elementary constituents of this puzzle are the two genders, the basic "Now"

numbers from which all else can be constructed. The two genders articulate the two fundamental dispositions. The generic geometries built from these dispositions articulate the generic situations and their corresponding dispositional geometry. Described in words, the project might sound fanciful. However, with the material presented, we introduce a new level of tractability into what has often been a very fanciful adventure over the last few thousand years.

According to Leibniz, the geometry of situations would simply explain many things, including plants and animals, with just a few letters from the alphabet. We *Now* know that the genetic code codes plants and animals with only a few letters. What we did *not* know is that these four letters of the genetic code also code the dispositional geometry of biological organisms. This is an important contribution of this paper. The way is *Now* open to really understand the genetic code, not just in terms of its ability to label amino acids but in terms of its semantics – what things *mean*.

<b><i>Instances of the Four Binary Generic Types</i></b>				
Gender	<i>mf</i>	<i>fm</i>	<i>ff</i>	<i>mm</i>
Syllogistic Terms typed Distributed Undistributed	<i>DU</i>	<i>UD</i>	<i>UU</i>	<i>DD</i>
Scholastic Syllogistic lettering of terms	<i>A</i>	<i>O</i>	<i>I</i>	<i>E</i>
Genetic Code Bases (RNA)	<i>a</i>	<i>g</i>	<i>u</i>	<i>c</i>
Stoic Binary Types	active-feminine	passive-masculine	passive-feminine	active-masculine
Stoic Indemonstrable Syllogisms	<i>4th</i>	<i>5th</i>	<i>2nd</i>	<i>1st</i>
The Classical Elements	Air	Water	Earth	Fire
Generic morphisms	monomorphism	epimorphism	bimorphism	isomorphism
Spacetime Geometry Lines	timelike	spacelike	lightlike	singular
Hyper-Complex Numbers	imaginary number $j$ $j^2 = -1$	imaginary number $i$ $i^2 = -1$	imaginary number $k$ $k^2 = 0$	“real”
Physics Force	Electro-Strong	Electro-Weak	Gravity	Strong Nuclear
The One and the Many	Oneness as Manyness	Manyness as Oneness	Manyness as Manyness	Oneness as Oneness

**Figure 9** There are four generic types in the form of the binary genders *mf*, *fm*, *ff*, and *mm*. Any organism looked at as a whole from a certain point of view will display instances of these four types. Examples range from the totally generic gender to Aristotelean and Stoic logic, complex numbers, spacetime geometry and even the four forces of physics. The four types are not abstract generalisations of *something* but generic universals of *anything* (any organism satisfying First Classness). The row for the four fundamental forces in physics has been added at the last moment as an ambit claim.

If the Stoics are to be believed, this kind of approach should not stop at the biological. An organism such as the universal, for example, should also be subject to the same organisational principles and indeed, the same code. In this case, there will be no difference between the coding entities and what is coded. The actual elementary particles and the codons that code them, will be the same thing. That train of reasoning will be pursued in another place.

It is the author’s opinion that it is time to put to rest the war between the two hemispheres of thinking, the age-old war between the analytic and the synthetic. For Charles Sanders Peirce, the two rival philosophical camps of his time were the Philosophy of Common-Sense and Critical Philosophy. He remarked that these two opposed ways of thinking were “at internecine war, impacificable.” A great change in consciousness occurs if one realises that

this very struggle is an unfolding process that takes place in everybody's head at all times of the day. According to the author, the paradigms of left side science and right side science are also the basic two paradigms for the organisation of consciousness. Once we know the underlying principles at play, a new Enlightenment becomes possible. It is hoped that this paper is a contribution to that noble end.

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