

The Universal Geometric Algebra of Nature: Realising Leibniz's Dream

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Abstract

Many researchers in the field of Geometric Algebra claim it to be the universal language of mathematics and physics and so realise Leibniz's vision. Their claim has some merit. However, Leibniz's geometry without number vision was much more ambitious. For example, Leibniz claimed that the geometry would explain the forms of plants and animals 'in a few letters.' Since the discovery of the genetic code, we now know such a code indeed does exist. However, Geometric Algebra makes no reference to the genetic code, the truly authentic algebra of Nature. In this paper we work closely with Leibniz's core concepts, together with a strong Stoicism influence, and show how his approach to metaphysics and geometric algebra can be made tractable. The end result is a geometric algebra based on a four-letter alphabet. The four letters are shown to correspond to the geometric semantics of timelike, lightlike, spacelike, and singular vectors of a geometric algebra. We propose an exact mapping of these four generic letters to those of the genetic code. Thus, the genetic code is interpreted as expressing generic geometric organisation based on a Leibniz style geometric algebra rather than being a mere transcription code as stated in the Central Dogma of biochemistry..

The overall picture presented here is that of metaphysics playing the role of an operational science much like Heaviside's Operational Calculus does in providing the much simpler synchronic alternative to the traditional analysis of time series. Like Leibniz, we claim that this new operational approach to science and philosophy can explain not just the generic forms of the animate, but also the inanimate.

Keywords:

quantum algebra, genetic code, spacetime geometry, metaphysics, geometric algebra, adjoints, first classness

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1 Introduction

Throughout his life, Leibniz worked on a project to construct a new geometry that would challenge the notion of passive extension implicit in the geometries of Euclid and Descartes. A new notion of space was called for. Instead of space playing the role of container for things that live in the space, Leibniz appealed for a non-dualist solution. Space and the thing become an organic unity and passive extension gives way to *organised* extension. Geometry becomes a theory of the self-organising machine. The project went under many names ranging from *Calculus of Situations*, *Characteristic Geometrica*, to *Analysis Situs*. Leibniz never published on the subject but his intentions in his *Analysis Situs* project of developing a geometric algebra without number was widely known. Leibniz stated his vision for a geometric algebra in a famous passage:

If it were completed in the way in which I think of it, one could carry out the description of a machine, no matter how complicated, in characters which would be merely the letters of the alphabet, and so provide the mind with a method of knowing the machine and all its parts, their motion and use, distinctly and easily without the use of any figures or models and without the need of imagination. Yet the figure would inevitably be present to the mind whenever one wishes to interpret the characters. One could also give exact descriptions of natural things by means of it, such, for example, as the structure of plants and animals. (Leibniz, 1969 (1956))

Leibniz presents a geometric algebra vision for understanding the structure of “natural things,” The key tenets of Leibniz’s vision can be paraphrased as a three-point hypothesis, which we will call the *Leibniz Hypothesis*. The hypothesis can be stated as follows.

- a) The organisation of biological life forms (“plants and animals”) can be explained by a simple algebra consisting of a few letters.
- b) The semantics of the algebra can be understood in terms of elementary generic geometric forms that explain the organisation of “the machine” and explain the motion and function of its parts.
- c) The same algebra is applicable, not just to coding biological life forms, but to “all natural processes,” which would include the organisation of matter.

From modern biochemistry, we now know that part (a) of the Leibniz hypothesis is valid. In terms of the RNA encoding of the genetic code, Leibniz’s “few letters” are {*a, u, g, c*}. In the genes and chromosomes of the genetic code, these four letters come in triplets, called *codons*, which transcribe to the twenty amino acid protein building blocks of all living organisms. However, the current biochemical interpretation of the genetic code as stated on its Central Dogma only sees the genetic code’s role as a transcription mechanism. The Central Dogma of biochemistry comes into conflict with part (b) of the Leibniz hypothesis. Instead of the four-letter code playing a simple labelling transcription role, the code becomes a vehicle expressing organisational semantics of each organism so coded. Our intention in this paper is to show that the four-letter genetic code articulates fundamental generic geometric forms and their dispositional semantics. We then move on to consider point (c) of the hypothesis.

We know that the genetic code applies to all biological life forms or to what we will call *animates*. Animates are subject to constant evolutionary change whilst the genetic code stays the same across billions of years. The code is not just there since the beginning, but seems always to be there *at* the beginning. A distinguishing characteristic of animates is that the genetic information is encoded in a genetic substance distinct from the functional substance, the two being related by a transcription process. In eukaryotes, the genetic material is enclosed in a membrane bound nucleus, for example. Moreover, there are two transcription stages, one isomorphic transcription from DNA to RNA encoding, and one from RNA to the amino acid building blocks of proteins. The less evolved prokaryotes lack a nucleus membrane isolating the genetic material from other cell body constituents. For some organisms like very primitive bacteria, there is only one transcription process with no DNA but just RNA transcribing to the amino acids. We establish the rule that to qualify as animate, the organism must have at least one transcription stage.

According to point (c) of the Leibniz hypothesis, the envisaged geometric coding algebra would not be limited to the biological. Biological matter can be thought of as *managed matter*, biological organic matter

managed by the organism concerned. For Leibniz, all matter that exists must be managed matter. We will refer to these non-biological forms of managed matter as *inanimates*. An example of an inanimate is the universe that we are a part of. The universe, as an organism, is responsible for organising the matter from which it is constituted. If point (c) of the Leibniz hypothesis is valid, then inanimates would be organised by the same universal geometric algebra as for animates. However, from what we know from the physics of matter, unlike animates, for inanimates there is no apparent distinction between genetic coding material and coded functional material. They must all be part and parcel of the same stuff.

In this paper, we very briefly consider that the elementary constituents of physical matter can be spelt out in a four-letter alphabet in accordance with point (c). Each letter of this four-letter alphabet possesses the semantics of physical force. There are four elementary kind of force, and these match up with the four known elementary forces in Particle Physics. Unlike in Physics, these forces and their generic geometric properties are determined from pure first principles free of the need for any empirical assistance. The elementary particles of Nature are then constituted from triads or codons of the four letters of the code. Each letter of the code corresponds to a physical “quarklet.” We claim that the quarks, leptons, and bosons can be interpreted as triads of quarklets, revealing that these “point-like” entities do indeed have internal structure. This leads to an alternative Generic Model to the Standard Model. Relying on simple computer generated database calculations an accompanying paper demonstrates exact fits between the Standard Model and a coherent subset of the Generic Model.

2 Demystifying Metaphysics

We must point out that this paper is not an attempt at a scholarly examination of Leibniz’s philosophy and writings. Some scholars may even object that the Leibniz concepts we describe here do not correspond perfectly with the writings of the historical Leibniz. However, **xy** following Leibniz to the letter, with all his twists and turns, **w**ould only would make our already challenging task more difficult. This work is not a commentary on Leibniz’s metaphysics. Rather we attempt to drag the key principles of Leibniz’s metaphysics onto central stage and make the whole thing scientifically tractable. The process in accomplishing this objective will be one of *demystification*. The objective of this paper is to demystify Leibniz’s scientific paradigm.

The first candidate for demystification concerns what is meant by metaphysics. According to Aristotle, the traditional sciences were characterised by their object of study having a determined genus whereas the object of study for metaphysics had undetermined genus. This means that the traditional sciences are categorical in nature and metaphysics, free of determined genus, must be non-categorical and universally unlimited in scope. For Kant, the object of study became the totally unqualified “thing in itself” that must be free of any determined *a priori* determinations.

In our Now Machines paper (Author, 2013a) (henceforth [NM]) we point out how the traditional sciences specialise in explaining the *a posteriori* in terms of the *a priori*. For example, formal mathematics deduces theorems departing from axioms established *a priori*. We informally referred to these disciplines as *left side sciences*. We pointed out that the alternative approach, exemplified by the Stoics, concentrated on studying the world in between the *a priori* and the *a posteriori*, the world that exists *now* - relative to the organism in question. We referred to this kind of science as *right side science*. For the Stoics, only corporeal bodies with extension exist. These bodies are restricted in a very important way. They cannot exist in the past, nor can they exist in the future. This was promoted as of quite some comfort to the Stoics as things from the past or future cannot hurt you, namely because they simply do not exist. Thus, the Stoic perspective is that objective reality is sandwiched between the *a priori* and the *a posteriori*. everything else bathes in subjective illusion. The study of such a reality is what can rightfully be called metaphysics. Leibniz held a similar view but with a non-materialist slant.

A good way of demystifying metaphysics seen in these terms is to realise that present day mathematics already contains examples of what we could call *weak metaphysics*. Take Mathematical Analysis for example, applied to the study of time series. A dynamical behaviour of a system can be described in terms of differential and integral equations. The input and output of the system would be functions of time. Until Heaviside came along, Analysis was the only way of describing dynamical systems. Heaviside applied “weak metaphysics” to the problem by converting this diachronic way of viewing reality to a *synchronic* view in the form of his Operational Calculus (OC). In this weakly metaphysical approach, the system inputs, the outputs, and the system behaviour itself, were all treated in exactly the same way as functions of a complex variable. Heaviside transformed complicated mathematics into simple algebra based on *multiplication*. Using functions of a complex variable, the output of a system $S(s)$ with input $F(s)$ is easily calculated by multiplying the two functions together. Heaviside showed that there are two perspectives for understanding a dynamical system, the diachronic “time domain” and the synchronic “frequency domain” of the Laplace and Fourier transforms of OC. System engineers are trained to think in both domains, changing hats when necessary. In the wider domain, the sciences still only think with one hat.

Heaviside's OC demonstrates a few key features of the synchronic take on reality. First of all the approach leads to great simplification by replacing complicated mathematics with a simple algebraic approach based on multiplication. Another feature is that there is no real distinction between **verb-like** and noun-like entities. In the OC, all entities are described in an identical way as functions of a complex variable. In brief, the operational "metaphysical" way of understanding systems is algebraic, multiplicative, and non-categorical.

However, what is lacking in the Fourier transform approach of the OC is that it is devoid of any **nowness concept**; in the quest to get rid of the "before" and "after," any semblance of a *Now* has been integrated out. Thus, the object of study has no interaction with the any individual subject. Consequentially, the kind of reality described by a Fourier Transform is realist and could even be called "mind independent." A metaphysical anti-realist flavour can be added by resorting to the Short Term Fourier Transform where the Fourier transform is calculated for a sliding window. The sliding window provides a rough and ready notion of nowness.

Of considerable interest is that the size and shape of the sliding window contributes significantly to the Short Term Fourier Transform result. In some cases the result might depend more on the specifics of the measurement window rather than the time function itself; a good example of subject-object interference. This interaction can even be expressed in an analogous way to Heisenberg's Uncertainty Principle (Oliveira & Barroso, 2003). In this context, the sliding window can be imagined as the *Now* of the subject, with the object corresponding to the signal. Both are present in the one window, in the one moment. This useful anti-realist illustration helps demystify what we mean by the formalisation of a subject-object relationship. It is interesting to note that as the sliding window is made increasingly narrower, the Short Term Fourier transform of any time series becomes independent of the time series and approaches the Fourier transform of the sliding window. This simply illustrates a fundamental aspect of metaphysics. Metaphysics is the science of the subject. Instead of looking through a window, we look *at* the window or even become a part of it.

Another weak metaphysics example can be found in field of study directly inspired by Leibniz's demand for a geometric algebra without number. Moving forward from Leibniz's time, the challenge of developing such a geometric algebra remerged in a competition set up in to celebrate the bicentenary of Leibniz's birth in Leipzig. Encouraged by Mobius, Hermann Grassmann put himself forward as the sole entrant and won the contest. However, Grassmann's approach did employ numbers but had the redeeming feature of not relying on coordinates. Grassmann's pioneering work, together with Hamilton's quaternions and the algebras of Clifford eventually became known today as Geometric Algebra (henceforth GA). In modern times, David Hestenes has shown how GA can be applied in a simplifying way to provide new insight into a wide range of physical topics from classical mechanics and electromagnetism, to Quantum Mechanics and Gauge Theory. He claims that the GA approach provides a universal language for all physics (Hestenes, 1988), a claim endorsed by others in the field (Doran, et al., 2002) (Li, 2008). GA is claimed to be a simpler and more fundamental approach to geometry than the prevailing coordinate based linear algebra Hilbert Space approach, which also has its roots in Grassmann's work on oriented numbers.

GA demonstrates similar metaphysical traits as for the OC. GA is simple and simplifying, which explains its widespread use in Computer Graphics applications. The approach is algebraic and multiplicative. In traditional vector algebra, geometric entities are point sets and form the nouns of the system. Transformations, in the form of matrices and tensor, make up the verbs of the geometry and are not geometric entities. In GA, there is no dichotomy between nouns and **verbs**; **all** are geometric entities. Transformations are replaced by the geometric product of one geometric entity with another. Thus, as well as being algebraic and multiplicative, GA is also non-categorical. However, in its present form GA presents a realist, subject independent reality. This is not the way that Leibniz would have imagined it.

We argue that the GA claim to universality is premature. Leibniz's project was much more ambitious and vast in scope as can be seen from his vision statement. GA does not address any of the three points of Leibniz's hypothesis. It does not even result in a geometry without number but merely a geometry without coordinates. As far as claiming to be the universal algebra of Nature, GA makes no reference to what is incontestably the prime candidate for such an illustrious role. Nowhere has GA a place for the genetic code, the truly authentic algebra of Nature. In this paper, we show how GA can be reengineered to answer all three points of Leibniz's vision and so integrate Nature's genetic code into its rightful place.

2.1 The First Principle

Every science must be based on some kind of normative principle as its source of truth. The normative principle of axiomatic mathematics is founded in its axioms assumed true. The empirical sciences are founded on empirical measurements, all assumed to be methodically harvested in a repeatable manner. These are all left side sciences. Our science is right side science and studies the entity without determined genus. This is the metaphysics of any-entity-whatsoever. The age old question is: What is the first principle for this universal science of practically anything whatsoever? This mind-boggling question becomes even more challenging when we demand that the principle must lead to a tractable science.

Aristotle had his answer in terms of his Prime Mover argument. However, it is the Stoics that we turn to for our answer to the question. We repeat the question in the form: “What is the first principle from which everything else follows?” The Stoics were impressed with the standard Cynic response. For the Cynic, there can be no first principle from which everything else follows. After answering the question in the negative, the Cynic ambles off to more fertile fields of endeavour to do what Cynics do. Not so the Stoic. The Stoic sees the Cynic response as a declaration of absolute truth. Here we find the most universal and profound law of Nature: There can be no principle from which everything else follows: *That is the principle*. In [NM] the requirement is termed the *First Classness Principle* (FC), a term borrowed from Computer Science.

We see this self-justifying, self-refuting principle of FC as the driving force for any self-organising system or organism. The principle interdicts any *a priori* entity from objective existence, be the entity a temporal precedent or a logical antecedent. Such entities cannot act upon or be acted upon. Only entities that exist in the systemic Now can act or be acted upon. The same applies to that which is *a posteriori*. Objective existence is the meat in the sandwich between the non-existent *a priori* and the *a posteriori*. The image that emerges is that of a Now Machine with all its membranes, branes, and folds of organisation, in synchronic coherence with the systemic Now. Some constituents are in total synchrony with the organism’s nowness. These are considered to enjoy real existence, relative to the organism as a whole. Other entities may be in partial synchrony. These are the phenomenal entities and can be considered as *imaginary*. As we shall show, the organism looked at as a whole from any particular point of view will appear as a complex entity having a real and an imaginary part. Now Machines are constituted from elementary complex entities and composites of such entities.

As we shall see, all of these entities are based on a Three-plus-One organisation with one real part and three imaginary. Our intention is to show that at the elementary level, the triadic part of the Three-plus-One structure corresponds to a triad of letters from the four-letter alphabet of the genetic code. Thus, we find ourselves back with an Operational Calculus of life, like Heaviside but on a much grander stage. Our aim is to show that the Operational Calculus of life and its semantics can be spelt out in the genetic code.

Leibniz had his own version of FC that he referred to as “the way of predefined harmony.” He found some tractability for the concept based on a Mini-Max principle of perfection. As Leibniz describes it: “God has chosen the most perfect world, that is, the one which is at the same time the simplest in hypotheses and the richest in phenomena.” The Mini-Max principle provides an intuitive way of understanding the FC principle. To achieve “the richest in phenomena,” the FC condition demands the absolute maximum of possibility. This can be achieved by demanding that the system be totally unconstrained. Turning to the Min part of the Min/Max principle, the simplest and minimalist constraint on such a system is that the system actually performs in such a way that it never falls under the spell of any extrinsic influence. The system must not only possess freedom from the extrinsic it must constantly struggle to maintain this freedom.

A popular subject for Leibniz scholars is Leibniz’s doctrinal stance on “extrinsic denominations.” Plaisted in his book on the subject wrote:

The philosopher Gottfried Leibniz said that one of his most important doctrines, and in fact one of the most important doctrines in all of philosophy as well as theology, is that there are no purely extrinsic denominations (Plaisted , 2002)

Understanding Leibniz’s stance on the extrinsic can be greatly demystified by realising that God’s perfect world requires the interdiction of extrinsic determinations, the interdiction being equivalent to demanding FC.

The demand of FC provides the universal normative apparatus for distinguishing the extrinsic from the systemic intrinsic. In the case of a biological organism one can think of the extrinsic in terms of systemic threats such as a bacterial or viral infection, cancer, or a debilitating injury, for example, The organism must be capable of distinguishing the good from the bad, discriminating between what is intrinsic and what is or has become extrinsic. The universal response is to maintain FC and thus re-establish systemic integrity. Achieving and maintaining FC expresses the generic purpose of any living organism, no matter what the scale, no matter what substrate, no matter if there is an intermediary transcription process or not.

The author proposes FC as the foundation principle of the operational science that we seek to develop. The supposition is that, starting from the demands of not violating FC alone, our operational science can bootstrap itself into existence. Note that the non-violation nature of the FC condition implies a kind of moral, ethical flavour to the approach. This was a point not lost to the Stoics. This kind of science has a moral imperative right at its core. What does not violate FC is good, what does violate FC is bad. Good or bad for whom or for what? It must be good or bad for the organism in question. These “moral” judgments are intrinsic, never extrinsic.

The emphasis in this paper is on the geometrical nature of the operational science. Before moving on, we should provide some tangible evidence that the FC principle is potentially tractable and not just a metaphysical thought bubble. Thus, what is a simple geometrical illustration of the moral imperatives of FC in action? One simple example is the traditional Cartesian representation of Euclidean space. The 3-D space is not admissible in our operational science because it violates FC. One culprit is the point at the origin, which becomes what David Hestenes calls, a *distinguished point*. The origin is the only point in the space that has zero displacement thus

established an absolute dichotomy between the origin point and all other points of the system, a clear violation of FC. The technical term is that the Euclidean space is not homogeneous. Another distinguished point, which is noticeable by its absence, is the point at infinity. These two violations of FC can be avoided by extending 3-D Euclidean space to a 5D projection space to form a Conformal Geometric Algebra (CGA) as described in (Hestenes, 2001). In the CGA representation of Euclidean space, all points are zero distance from the generic origin and also zero distance to the point at infinity. The generic origin and point at infinity are no longer distinguished points, but each a point just like any other. FC violation is avoided. CGA, because of its simplicity and expressive power, is widely used in Computer Graphics (Dorst, et al., 2007).

3 Leibniz's Geometry of Situations

Leibniz sums up geometry: "In total, there are two things to study in geometry, extension, and situation." However, Leibniz avoided any precise definition of what he meant by situation. As (Risi, 2007) remarks:

Once again, however, any enthusiasm we may have for definitions will be **disappointed**. as there is no way to find even the ghost of a definition of the situation concept throughout Leibniz's writings.

In Cartesian geometry, the notion of a situation might be interpreted as a location in space. Spatial location can simply be formalised in terms of coordinates. A location becomes simply a point in space that can be known by its attributes expressed as coordinates. However, these coordinate-attributes are purely accidental as they depend on some arbitrarily chosen reference frame imposed extrinsically. Moreover, the construct establishes a rigid dichotomy between geometric points as first class geometric entities and numerical coordinates that are not geometrical entities and hence second class. This violates FC. Cartesian geometry is incompatible with FC. The geometry that is compatible with FC must be of another kind, a geometry without coordinates just as Leibniz demanded.

A coordinate system is just one example of all of the externally imposed attribute systems that abound in the traditional sciences. Attribute based systems are all categorical in nature. Underlying such systems is the mathematics of Set Theory. Aristotle summed it up by noting that all such sciences studied entities with a determined genus. The other kind of science, metaphysics, must be free of determined genus. Instead of an attribute-based science, the science must be attribute-free. For Leibniz the science would be free of extrinsic denominations. All denominations would be intrinsic and relational in some way. The purpose of this paper is to show the way that this can be accomplished.

Instead of developing a categorical attribute based science, we develop a non-categorical methodology for individuation of entities. Instead of attributes, we will rely on an ancient construct that we will call *ontological gender*. In keeping with the geometric thrust of this work, we introduce the gender interpretation of Leibniz's two key constituents of geometry, extension and situation.

3.1 Ontological Gender

The object of study will be something that exists as a pure body of extension totally unqualified. We will say that such a geometric entity will be of *feminine gender*. The specificity of the purely feminine entity is that it has undetermined genus. This total lack of determined specificity is nevertheless, a specificity. Other entities have determined genus, the feminine entity has *undetermined genus*. Thus, it can be said that the pure feminine entity at least has *something*. In order not to violate FC, this something must also share the same existential status as the feminine entity. Thus, the feminine entity *has something*, a specificity albeit totally unqualified. This something exists and will be said to be of *masculine gender*. The feminine *has undetermined genus*, the masculine *is that undetermined genus* as an entity in its own right. The feminine *has it*, the masculine *is it*.

Thus, the object of study consists of two entities, one feminine, and one masculine. These two entities are different as they differ by gender. This is the view from within the system, the view from the perspective of the masculine entity. Viewed from outside the system, the masculine and feminine entities appear indistinguishable, as there is no way of distinguishing the two based on a difference of specificity. There is only one specificity to go **around**; the feminine entity has it, the masculine entity is it. In this perspective, the two entities appear in *superposition*.

We aim to show that the gender construct can explain the richest in phenomena apart from the simplest of hypotheses. This is Leibniz's Min/Max principle in action. We intend to explain the phenomena of having and being by using a two-letter alphabet composed of *F* and *M*. An indication of how this is going to work can be found in **Socrates'** confession of ignorance. Socrates confessed that the only thing he knew with absolute certainty was that he knew nothing with absolute certainty. Here, the certainty clause applies to the masculine, an assertion of knowledge. The uncertainty clause applies to the feminine. This Socratic formula expresses **the our modus operandi**. In the resultant *M* and *F* based algebra, *M* will express what is known and *F* will be carried along in the algebra as the universal wildcard.

3.2 Leibniz Notation

Leibniz's declared objective was to express completely a geometric figure using only letters, without any verbal aid or diagrams. Influenced by the ancient Greek geometers he explored the opposition between the known and the unknown to generate loci. His labelling system denoted known points by the letters A , B , C , and unknown points by X , Y , Z . Thus a line, as extension, could be defined by the couplet of letters AB . Any point between the ends of the line could be talked about in terms of the Y in the term AYB . Leibniz explored different geometric relations between figures such as identity, congruence, equality, and likeness or similarity. In this paper, we examine likeness relations and use the symbol γ .

Leibniz gives some simplistic examples of using his notation. The term $A\gamma B$ means that the given point A is *like* a given point B . The term AB denotes a line with known points A and B and hence of known length and angle. The line AY in the expression $AB\gamma AY$ will be a line *like* the line AB sharing the common point A . If likeness is defined as equality of length, and Y denotes all the possible Y points, then $AB\gamma AY$ proscribes the surface of a sphere. In the same way, the expression $AX\gamma XB$ would describe a plane halfway between A and B . X would be all the points on the plane.

Leibniz's geometry never really got off the ground and so came nowhere near implementing the richness of his metaphysics. One limitation was that all his development took place within the confines of an assumed Euclidean geometry. A first weakening of this unnecessary assumption is to allow directionality into the geometry. In this case, the AB and the BA in $AB\gamma BA$ may no longer be the same geometric entity. Considering this, it quickly becomes apparent that the simplest and most direct way of cutting to the chase is to go all out and completely revamp Leibniz's vision in a more powerful environment. We will thus recast Leibniz's vision and intent in the language of the gender typing. Generic Geometry will thus become, not just a labelled geometry, but also a *gendered geometry*.

Ontological gender provides a ready-made lettering system for the algebra, consisting of the two-lettered alphabet $\{F, M\}$. The next step in developing Leibniz's universal geometric algebra is to provide the geometric interpretation of the two gender types F and M . First of all, the F typed entity can be interpreted as a geometric extension totally lacking in determined specificity. One can say that **that** the F type entity corresponds to content (as extension) without determined form. Form can informally be thought of as corresponding to the *situs* of the various parts of the F entity. However, F is not known to have any parts and so form can only be expressed in terms of the *situs* of F itself. The difference between an F type entity and an M type entity is that the latter involves a more determined *situs* than that of F . The challenge is to develop a geometry that formalises this difference and constructively builds upon it.

If one were ever capable of relocating oneself to the situation marked out by M , one would have a very privileged and fundamental point of view for examining F . This situation can best be described as the *God's eye* view. It is only accessible to that breed of entity that enjoys non-determined *situs*. It can also be called the *objective* point of view and has sometimes been referred to as the *view from nowhere*. This point of view is the one adopted by the classical sciences, axiomatic mathematics, and analytic philosophy. The methodology for formalising science based on this point of view is very simple: just ignore the M entity and declare the science as being *mind-independent*. All these sciences can declare themselves objective and based on one single point of view, albeit totally undetermined. Such science cannot directly study F because such an enterprise would be intractable due to F having totally undetermined genus. Such sciences only study X where X labels the particular object of study of the specialised science in question, the choice of X being subjective.

Now a fundamental requirement demanded by Leibniz is that his approach to the science must embrace reality as it appears to us and not just to the eye of God. In the geometric scenario, this requirement translates to individual geometric entities with individual form. Once again, we invoke the gender-typing construct. This time we start with the geometric body with no determined specificity except that of having individuality. We will say that this entity possesses feminine gender but, to distinguish it from the impersonal, non-individual case, it will be denoted by the lowercase letter **f**. The principle of FC demands that this specificity that *f* has, that of individuality, must be an entity in its own right. We will say that this entity does not *have* individuality but rather *is* individuality. It will be said to possess masculine gender and denoted by the lowercase letter **m**. The entity **m** corresponds to the form of the extension **f**. In other words, we can say that the individual extension **f** is characterised by having a determined *situs* **m**. This is in addition to the non-individual extension F characterised by having undetermined *situs* M .

In [NM], the author claims that these two pairs of simply gendered entities correspond to the Stoic Categories. The F and M entities correspond to impersonal entity and quality respectively. The **f** and **m** pair corresponds to what the Stoics called disposition and relative disposition. If our science were to be a traditional categorical discipline, these elementary categories would be combined additively. However, our science must be dispositional. These "categories" are treated as prototypical geometric entities that combine to produce

qualitatively more complex organisation. Instead of combining additively, they combine *multiplicatively*. The product of the two simply gendered pairs to construct four binary gendered pairs will give:

$$\begin{Bmatrix} m \\ f \end{Bmatrix} \otimes \{F \quad M\} = \begin{Bmatrix} mF & mM \\ fF & fM \end{Bmatrix} \quad (1)$$

The semantics of the product will be explored further on. The product is analogous to the geometric product of Geometric Algebra pioneered by Grassmann where the product produces geometric entities of higher “grade” than the constituents. Intuitively the semantics of the product is illustrated in Fig. 1 where the result can be interpreted as four gendered types of substance, an expression of the four classical elements.

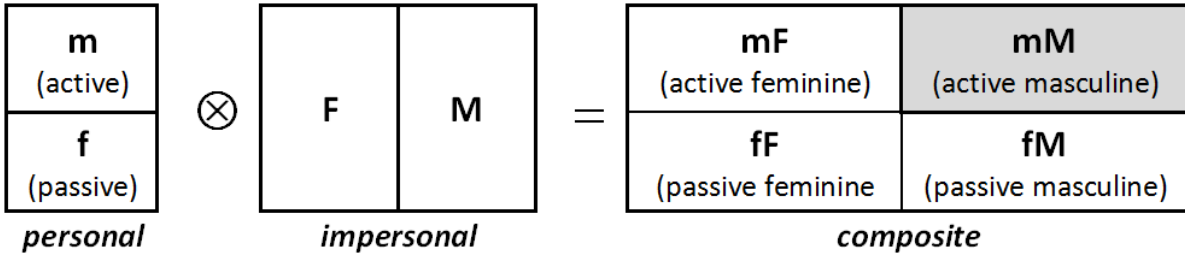


Fig. 1. Illustrating the product of personal and impersonal genders to construct the four binary gendered types (Doug, why is mM shaded & not any other binary?)

The Stoic system included the two impersonal genders as shown here but the Stoics referred to the personal masculine and feminine genders as the active and passive principles respectively. This ties up with their typing of the four classical elements air, earth, water, and fire as generic substances typed as active F , passive F , passive M , and active M respectively. As explained by the author (Author, 2013a), the same typing is implicit in four of the five indemonstrable syllogisms of Stoic Logic. In other words, Stoic Logic could be interpreted as the Logos of generic substances, as well as its interpretation in terms of propositional logic.

3.3 The Objective Starting Point

The starting point question has already been discussed in the previous paper. Here we briefly provide another account from the Generic Geometry perspective. Unlike axiomatic mathematics, which is non-constructionist, the alternative non-dualist mathematics based on FC must be constructionist. Thus, the geometry we are developing here must be constructionist geometry. This introduces a very thorny question. To construct, there must be a starting point for the construction, preferably an *objective* starting point, in other words, an objective *situs*. The objective *situs* M was introduced above and intuitively interpreted as formalising the concept of *anywhere*. Starting absolutely anywhere at M might be a sound idea but absolutely impossible to implement in practice, at least for mere determined mortals like ourselves. Being, as we are, determined entities with determined *situs*, we have no choice but to settle for second best. We must start not *anywhere* but rather start *somewhere*. This means that we have no choice but to start with the *subjective* starting point in the form of the determined *situs* entity labelled m .

Our objective is to construct an objective science not a subjective one, even though the objective may include and subsume the subjective in some way. Starting with the subjectively determined *situs* m seems to compromise our objectivity aspirations. However, all is not lost if the geometry is such that these two *situs*, m and M , are indistinguishable from each other. Such a geometry would be *starting point invariant*. One can say that the FC principle even demands that this be the case. No matter where you start, you will always get the same theory, so to speak. The FC principle must be self-justifying. Of course, there is not much choice as, due to FC, there are no other principles hiding in the shadows.

The product operation introduced above, leads to four binary typed entities arranged in a kind of “semiotic square.” The individual masculine correspond to the “front lobes” of the square, the individual feminine to the back. The impersonal feminine corresponds to the left side and the impersonal masculine to the right. The relationship of this semiotic structure to the Aristotle Square of Oppositions, and to our reconstructed Stoic version, is discussed in our previous work and does not concern us here. What matters is the geometric interpretation of the four binary gender types $\{mF, fF, fM, mM\}$. The requirement of starting point invariance is that the determined *situs* m (the “accidental” starting point) is indistinguishable from the undetermined *situs* M (the universal, generic point).

Qualitatively, the geometry should be such that no matter what the accidental starting *situs* \mathbf{m} , it must appear from the point of view of \mathbf{m} that \mathbf{m} is located at the “centre of the universe,” so to speak. If the four binary types are interpreted geometrically, they become the four universal types of geometric line. If the line mM joins the two indistinguishable *situs* \mathbf{m} and M , then the length of mM must be either zero or infinitesimally small.

(Doug, does it matter that \mathbf{m} alone is in bold, but in mM is not? Only for *situs*?)

3.4 The Four Generic Line Types

Robert Goldblatt explored the four generic kinds of vector within an abstract axiomatic framework in his book *Orthogonality and Spacetime Geometry* (Goldblatt, 1987). the four vector types are the familiar, timelike, lightlike, spacelike, vectors with the addition of the seemingly degenerate singular vectors. These vector types correspond to the generic gendered vector types mF , fF , fM , and mM respectively. Thus, an mM type vector in Generic Geometry is analogous to the singular vector. From a traditional mathematical perspective, singular vectors are orthogonal to all other lines present including themselves. They appear so degenerate that axiomatic mathematics ignores them. However, in the non-Cartesian paradigm of Generic Geometry, singular vectors in the generic guise of mM type vectors play a pivotal role (literally) as will be seen.

Note that if the four binary genders are represented by dyads, then writing the genders in lowercase and majuscule (uppercase?) becomes superfluous. The first source part of the dyad will always correspond to personal gender whilst the target side will always be impersonal. Sometimes we will dispense with the lowercase-uppercase convention, but for the moment, we will keep using it so as to hammer home that the first part of binary gender is always the lowercase-labelled individual side of the equation.

The four generic vector types make up the four elementary geometric entities of Generic Geometry from which all the composite geometric forms of any system based on FC can be constructed. In [NM], it is argued that the composite forms must be based on a multiplicity of triads of dyads, where each dyad is one of the elementary binary gender typed entities. There are, of course, 64 possible combinations for each triad. Of crucial importance is that the triads must not violate FC and so no endpoint of a dyad can be definitively before or after the others. The only solution to this requirement is a triad, or multiplicity of triads, where the dyads are arranged in a “colour-coded” RGB configuration as shown in Fig. 2. The structure is self-labelling as what should be labelled R, G, or B, is uniquely determined by the geometric structure of the triad. The triad part of this Three-Plus-One structure can be intuitively thought of as representing the phenomenal and potentially observable, which in turn determines the unobservable “now” part of the structure.

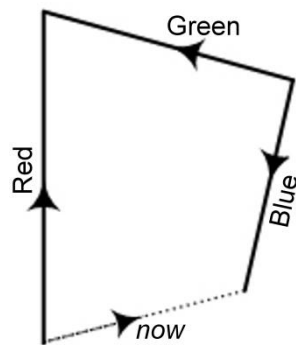


Fig. 2. The generic RGB triad structure. The “now” part is determined with the three dyads representing the “phenomenal” or “observable” aspect of the entity. The now part is never observable and can be **considered** “white” in colour. The RGB triads are placeholders for binary typed entities taken from the four-letter alphabet $\{mf, ff, mf, mm\}$. Thus the RGB triad configuration can be typed in 64 different ways.

Using the RGB triadic structure, we can construct a gender typed triad that solves our immediate problem of building a starting point invariant geometric structure. Starting with the mF type vectors (\mathbf{m} is the determined starting *situs*) as the R dyad of the RGB triad, we end up with the Three-Plus-One structure illustrated on the right side of Fig. 2. The right side of the diagram illustrates a triadic structure made from mF , fF , and fM dyads, which combine to determine an mM dyad. If the mM dyad is assumed to be of zero length, and the orientation of the lines is ignored, the structure resembles a standard spacetime diagram made up of cones of timelike and spacelike lines, together with a bundle of lightlike lines.

There are four generic types of dyad labelled by the alphabet $\{mF, fF, fM, mM\}$. For convenience, a single letter could label each of these four types. Throughout all of our work, we have endeavoured not to invent new fancy terminology but to rely on the historically tried and tested. We are innovation averse for most things, including our terminology. Paraphrasing Charles Sanders Peirce, when it comes to fundamental ideas innovation has little to recommend it. We even balk at inventing a new four-letter alphabet from scratch. One four-letter

alphabet with a long pedigree consists of the letters $\{A, I, O, E\}$ as used by the Scholastics to label the four terms of the syllogistic. It was shown in a previous paper how this alphabet, and its implicit distributed and undistributed typing, maps to gender typing and so would be a good choice of letters. However, we have chosen to be more brazen and opt for a four-letter alphabet of even longer pedigree, the letters $\{a, u, g, c\}$ respectively, making up the RNA encoding of the genetic code.

In [NM] we proposed the *Generic-Genetic Code Conjecture* that states that $\{a, u, g, c\}$ maps perfectly to the gender typing $\{MF, FF, FM, MM\}$, and in that order. In this paper, we assume that the conjecture is valid. Thus, we continue to use this four-letter labelling until such time that its validity were ever to be proven otherwise.

4 Organising Organised Matter

Leibniz's philosophy of what constitutes matter differs from that prevailing in the natural sciences of today. For Leibniz, matter in Nature had to be *organised* matter. Any entity is more than just a Cartesian extended body; it must be accompanied by its organising principle. In the case of a body consisting of a simple substance, Leibniz's organising constituent was in the form of an incorporeal automaton that he called a *monad*. Bodies were corporeal and could act and be acted upon, whilst the incorporeal monads could not.

Stoic philosophy also employs a similar opposition between the corporeal body and its accompanying incorporeal partner. For the Stoics, the incorporeal was called the *lekta* or 'sayable' which expressed the principle of how the corporeal substance could be understood as an object of comprehension. In Stoic physics, there were four simple substances, the four classical elements of Empedocles. The four elements of both Empedocles and the Stoics were considered to be gendered, even though there was no formal understanding of the gender construct. In previous work, we have shown how these gendered substances map up perfectly with four of the five indemonstrables of Stoic logic. Thus, the *mF* gendered Air element had for its *lekta*, the second Stoic indemonstrable; the *fF* gendered Earth, the fourth indemonstrable; the *fM* gendered Water, the fifth indemonstrable; and the *mM* typed Fire to the first indemonstrable. The Stoics also claimed a fifth substance called *pneuma*, which we claim would have the 'incompatibility' third indemonstrable as its logical *lekta*, thus completing the quintuple. The third syllogism expressed the principle of non-contradiction as a universal; an entity can have one quality (gender) or the other, but not at the same time. The third syllogism may be interpreted as the Stoic version of the principle of non-contradiction. Contradiction is possible and even inevitable but never at the same time.

The system of five foundational syllogisms was undoubtedly due to Chrysippus. His syllogistic structure provided Stoic physics with a formal basis that matter be animated by Logos rationality as originally claimed by Heraclitus. The Logos, in the form of *pneuma*, expressed itself through four kinds of Heraclitean flux, four kinds of force, each logically formalisable by a Stoic indemonstrable. There are four determined simple substances. These substances are corporeal bodies that can act and be acted upon. Each substance has its own characterising incorporeal *lekta*. From a logical perspective, each of the four *lekta* corresponds to one of the Stoic syllogisms. Stoic logic thus should be considered primarily as the *logic of substance*. We ignore here the secondary uses of Stoic logic as an ancient version of the propositional calculus applicable to everyday contingent reality. The interpretation of Stoic physics provided here was first proposed by the author in our previous two papers.

In this paper, we liken the Stoic *lekta* to Leibniz's monad concept. Leibniz did not succeed in formalising monads in any tractable way. However, his likely intention was through his *Analysis Situs* geometry. He also had some ideas concerning logic, which we can ignore for our purposes, as the Stoics seem already to have had that base covered well enough for our needs. Thus, Leibniz's monad can be considered as a geometric version of a Stoic *lekta*. Monads should allow us to talk about the simple substances geometrically. Monads are geometric 'Sayables.'

Monads also implicitly state a very fundamental aspect about a reality based on pure rationality, a reality based on FC. A prime condition for an entity to qualify for the status of rational existence is that it be sayable. The sayable construct demands a language in which the sayable can be expressed. As we keep insisting, the language will be in the form of a generic code of which the genetic code is an instance. No corporeal body can exist without being accompanied by its *lekta* monad expressed in this generic code.

According to Leibniz, monads cannot ever be present without an accompanying body. The body is corporeal, the monad is incorporeal. One should avoid the temptation to think of a monad as some kind of mystical ethereal figment of the imagination. Nevertheless, one should also avoid the temptation of trying to nail the monad down to some categorical determination. For example, is the monad energy or information? Neither energy nor information can directly act upon or be acted upon and so such intuitions can be tempting. We prefer to understand the monad as one side of an opposition. Just as the monad cannot be present without an accompanying body, neither can the understanding of a monad be embraced in isolation. Many possible oppositions suggest themselves such as that between mass and energy, a hardware-software opposition, or the two poles of a particle-wave duality. At this stage of the development, we prefer not to ground the body-monad

opposition to any specifics and so leave the opposition as generic. The monad is incorporeal and cannot act or be acted upon. However, the monad provides the power, the energy, the potentiality, for its accompanying body to act and be acted upon. The body provides the mechanics; the monad provides the mechanics with the power to act in a coherent integrity maintaining way – in accordance with the principle of FC, of course.

In discussing conservation laws, Leibniz went to pains to emphasise that the magnitudes involved were not scalars. In this context, the monad must have geometric form. The monad would express its dispositional capability as a geometric form, not as scalar quantity. For Leibniz, the union of body and monad became the union of Body and Soul for the more complex organisms like plants and animals. The first part of the opposition can act and be acted upon whilst the second, be it energy, information, software, or whatever, cannot. If we take the term soul as a technical term, there is nothing occult about it. Matter in Nature is organised in a particular way, not from laws imposed from outside but from within the organism itself, be it animate or inanimate. The entities of Nature are organised *sui juris*. Each entity in Nature is responsible for its own lot. The principle of such organisation is the ‘soul’ of the entity, not the body.

For the implacably materialist Stoics like Zeno, the founder, the soul was corporeal and material and so could act and be acted upon. From this dogmatic materialism, the non-duality of Nature would be a *fait accompli*, not something to be continually striven for. Of course Leibniz went in the other direction and promoted the supremacy of the incorporeal over the corporeal and so has been characterised as an idealist, the very opposite to the materialist Stoics. However, as we will see, when represented in terms of Generic Geometry, one will be able to illustrate that the organisational geometry of the ‘soul’ part of an entity is similar, but not identical to that of body geometry. One could say that soul is formally *like* matter, and *vice versa*. Let us now look at an example of a geometric body armed with its geometrical organisation, its ‘geometric soul’ so to speak, its monad.

None of the material in this section was mentioned in our previous work. Here, we claim to provide the rudiments of a geometric formalisation of what Leibniz was so excited about when he published *New System of the Nature of Substances and their Communication, and of the Union which Exists between the Soul and the Body* (Journal des savants, 27 June and 4 July 1695). Over three centuries later the current author, he too is excited.

4.1 Bridging Law between the Incorporeal and the Corporeal

By incorporeal, we mean the sayable ‘theory side’ of the equation. Theory substance, unlike corporal bodies, cannot act or be acted upon. In the Leibnizian paradigm, both substances, the monadic and the body, must be present and form part of a unified whole. There will be the incorporeal theory substance on one side and corporal body on the other. Mind meets Matter.

David Chalmers posed the often-cited ‘Hard Problem’ concerning the ‘high level bridging laws’ between mental processes and experienced physicality. In our case of generic geometry, we have an example of the ‘mental’ side in the form of the *aug* codon dictating the principle of starting point invariance. We now have to fill in the missing physical side. One way to do this is to apply the necessary ‘bridging law’ to impute the missing side. Seeing that both sides belong to the same geometric whole, the problem should offer a neat geometric bridging law solution. Since the mathematician God of Leibniz is at work here, we can assume that the solution will be neat, and above all simple whilst allowing for the maximum of possibilities. Thus if M denotes the monadic soul or monad and B denotes the body sides of the geometry, taking an operational approach, the geometric union of B and M will be defined by the product of B and M , i.e.,

$$BM \gamma I \tag{2}$$

The specifics of B and M have not yet determined, neither has the product, nor the like relation γ . However, if we make a preliminary interpretation that the product is the geometric product and that the like relation is equality, then $BM = I$. A conventional mathematical interpretation of (2) is that B is the left adjoint of M and M is the right adjoint of B . Can Chalmers bridging laws be explained in terms of left and right adjoints? The solution should be simple, but is this too simplistic? We should not be too concerned. Leibniz’s metaphysics was decidedly optimistic. Provided that our proposed solution leads to more possibilities than any other existing approach then we should be on the right track.

4.2 Bodies, Monads and Bra-Ket Notation

Generic Geometry involves the unity of two ‘half spaces’ making up a whole. One half-space corresponds to the corporeal body of an entity, the other half to the monad, the ‘mind’ side of the construct. The monad may be a simple, elementary entity or a composite structure. The composite monad will be called a monad pure and simple in order to emphasise its non-dualist vocation. Of course, the end result will be a rather lopsided organism with only half a mind and half a body. The solution is *bilateralisation*. The half-being must in turn be accompanied by its generic partner. One half-being will play a monadic role, the other half-being will play the

body role as illustrated in Fig. 3, where the right monad half-being would probably play the monadic role. But that is only from one perspective. From another perspective, it may be the converse with the left monad playing the dominant monadic role. In this paper, we ignore bilateralisation details and focus on the monad-body relationship.

The unity of monad and body is expressed as them being adjoints to each other. To express the construct symbolically we borrow the convenient *bra-ket* notation introduced by Paul Dirac to Quantum Mechanics in 1939. The *bra-ket* notation can be traced to Grassmann's notation for the inner product. In Quantum mechanics the quantum states of a system can be written in *ket* notation as $|\psi\rangle$, where ψ is the wavefunction as a vector in *ket* space. The 'collapsed' states of the system can be written in *bra* notation as $\langle\phi|$ where ϕ is a vector in *bra* space. The inner product of a *bra* $\langle\phi|$ and a *ket* entity $|\psi\rangle$ can be written as $\langle\phi|\psi\rangle$. Together with an operator P to determine measurables leading to the *bra-ket* notation $\langle\phi|P|\psi\rangle$, some of the essentials of Quantum Mechanics can be expressed in *bra-ket* notation. The end result is that the inner product form evaluates to a scalar value which can be interpreted as a measure of probability in some kind of space – the probability of a particle location or the probability of its momentum having a certain value, for example.

In this paper, we borrow the *bra-ket* notation and the fact that it illustrates a fundamental cleavage between two diametrically opposed aspects of reality. In our case, the fundamental oppositions between the organizing and the organised poles of organized matter. The organiser is the Leibniz monad, the organized is organised matter, the body side of the equation. It is here that we see the difference between this kind of physics and that of traditional science.

Employing our version of the *bra-ket* notation, a monad will be represented by the *ket* $|\Omega\rangle$ which denotes a spatio-geometric *ket* entity. A *ket* entity is characterised as an incorporeal spatio-geometric entity, which cannot act or be acted upon. Unlike in Quantum Mechanics the *ket* is not an entity *in a ket* space but rather that space *as* an entity in its own right. The idea is that the *ket* $|\Omega\rangle$ must escape the Cartesian container-contained dichotomy. Space and the thing-in-space must be the same thing. The *ket* $|\Omega\rangle$ thus establishes a monism for the incorporeal aspect of the construct in hand. The *ket* declares the first salvo of an *idealist monism*.

Our use of the *bra-ket* notation is to highlight the fact that every entity has two sides to it, a right *ket* side, and a left *bra* side. The *bra-ket* opposition is analogous to the opposition between placeholder and value. The analytic way of thinking of the traditional sciences tends to only deal with one side, usually value. In the synthetic way of reasoning of Generic Science, both sides must be present in the same moment. Thus, accompanying the non-corporeal *ket* side there must be a corporeal *bra* side denoted, the latter being denoted by $\langle\Sigma|$. Unlike the incorporeal *ket*, the materialist, corporeal *bra* entity has the power to act upon and be acted upon. Once again, the corporeal *bra* entity must escape from the Cartesian contained-container duality. The *bra* is the first salvo of a *materialist monism*.

The materialist Stoic would say that only the corporeal *bra* $\langle\Sigma|$ side of the opposition objectively exists. The *ket* side is an incorporeal lekta or 'Sayable,' a means for expressing the semantics and deep meaning implicit in the body $\langle\Sigma|$ side of the opposition. For the idealist inclined Leibniz, the *bra* $\langle\Sigma|$ body side of the opposition is secondary to the incorporeal *ket* $|\Omega\rangle$ side, the latter being identified as a monad. The *ket* $|\Omega\rangle$ could be interpreted as an embryonic expression of a monad and would correspond to the first glimmer of Mind. For Leibniz, it was Mind over Matter. To understand how this all works, one must express it all in terms of monads.

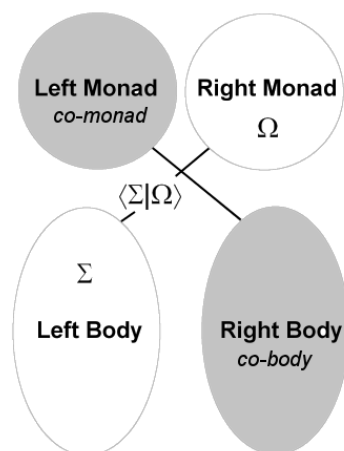


Fig. 3. Illustrating the two sided 'left handed' entity denoted by the *bra-ket* $\langle\Sigma|\Omega\rangle$. In order not to violate FC there must also exist a right handed version shown shaded.

There is no irreconcilable difference between the Stoic and Leibniz perspectives, only a subjective preferential emphasis. Moreover, this little storm in a teacup is overshadowed by a much more imposing conflict that could bring this whole construct tumbling down. Our *bra-ket* formulation establishes a chirality convention; the corporeal *bra* entity is on the left and the incorporeal *ket* entity is on the right. Here we are assuming that the syntactical order of the *bra-ket* notation corresponds to a physical ordering. Such an assumption is anathema to the analytic philosopher as it involves the classical sin of confusing use and mention. However, maintaining a dichotomy between use and mention only makes sense within a dualist paradigm. In the non-dualist paradigm of Generic Science, the rigid distinction between use and mention no longer prevails.¹

If the chirality implicit in the *bra-ket* notation were to be interpreted as a law claiming to state the way things actually are, then obviously, the construct violates FC. The absolute iron law of FC states that there are no absolute iron laws (the latter not negating the former). Thus, the *bra-ket* notation only articulates one kind of entity, an entity where the corporeal body side is on the left. We will refer to such entities as being *left-handed*. In order not to violate FC, as well as left-handed entities there must also be *right-handed* entities.

The left handed and right handed (shown shaded) version of the entity are illustrated in Fig. 3. Only the left-handed monad-body couplet is denoted by *bra-ket* notation. Thus, only the right monad is a true monad. The left monad will be referred to as a *co-monad* and its corresponding body a *co-body*. The diagram shown here is really too premature to have any reliable scientific value. However, it is suggestive of a natural disposition of monads to be arranged in such a *suggestive* bilateral structure. However, anything we say here might only apply to animates and not to inanimates.

The *bra-ket* $\langle \Sigma | \Omega \rangle$ plays two roles. First, it denotes the monad-body couplet and secondly it can be evaluated as an algebraic expression. As an algebraic expression, the couplet should signify the geometric product of the *bra* and *ket* geometric entities. Since the *bra* and *ket* should be “orthogonal,” the outer product part of the geometric product would be zero. The geometric product thus becomes the inner product, which is the same way that Quantum Mechanics treats the *bra-ket* construct. As an algebraic expression of a monism, the inner product should intuitively be unity. Thus, the evaluation of the *bra-ket* must give

$$\langle \Sigma | \Omega \rangle = 1 \tag{3}$$

In following sections, we look into the four ways of achieving this result, corresponding to four distinct types of elementary monad-body pairs.

In summary, the *ket* part of the construct articulates an *idealist monism*; the *bra* part articulates a *materialist monism*, whilst the evaluated combination as described in (3) articulates the organisational constraint for an *organised monism*. The organised monism integrates the idealist and materialsit standpoints.

5 The Four Elementary Geometric Forms and their Generic partners

As developed in our previous work, any system based on FC must be constituted from four elementary substances. The actual physical nature of these substances will depend on the system concerned. For example, what corresponds to substance for animates will be different *from* that of the inanimate. The same would apply if humans ever arrive at constructing artificial animates. What remains common is that the forms, the logic and geometry of the forms, all remain universal. In this way, we revive the ancient concept of the four classical elements. [NM] investigated relationships with Aristotelian and Stoic logics, spacetime geometry, and GA. The unifying concepts behind our analysis are based on gender. The two genders are not attributes like colour or sex. They are the two universal dispositions from which generic forms can be constructed. The only determined specificity of the four universal substances is that each is binary gendered.

In this paper, the four gendered substances are interpreted as four generic constituents of geometric form. Sometimes we will borrow a term used in String Theory, and refer to these elementary substances as *branes*. There are thus four universally generic types of brane. Each of the four different types of brane can be characterised by the characteristic of point pairs that have *situs* on the brane. Thus any point pair on an *MF* typed brane will have endpoints typed *M* and *F*, for example. Of particular interest is the *MM* typed brane. In this case, all point pairs will have determined *situs*. One could refer to the *MM* typed brane as representing the *now* of the organism where *situs* (*siti*?) on the brane are all in synchronic concurrence. The geometry of each

¹ (Mates, 1986) notes that the writings of Leibniz demonstrate his “carelessness about matters of use and mention; we get the impression that frequently he doesn’t care whether he is understood to be talking about sentences, propositions, or the corresponding states of affairs, or, again, about descriptions, concepts, or the individuals to which the descriptions, as expressing the concepts, refer.” For the universal non-dualist synthetic paradigm of Leibniz, the analytic distinction between use and mention is of no fundamental concern. The entities involved are generic and hence are devoid of determined genus. When talking about them, one is literally talking about “anything.”

type of brane can be studied in terms of vector dyads with endpoints on the brane. Thus, the study of the four types of brane becomes a study of the four types of vector. We will now look at each of the four elementary geometric **brane-like** substances corresponding to the four gendered binary types and their generic partners.

The fundamental message of Nature is that there are four elementary systemic constituents involved in any rationally organised system, the ultimate rationality being that imposed by the FC requirement of non-duality. There are four letters in the genetic code, there are four fundamental forces in physics, and there are four term types in Aristotle's syllogistic logic. In previous work, we have shown the four-fold structure implicit in Stoic logic and of course, there is the ancient theory of the four classical elements. In Category Theory, there are four types of morphism, epi, mono, iso, and bi. Category Theory has also come up with universal structures such as the pushout, the pullback, and limits but these will not be considered here. As one can see, the world of the four classical elements is a very crowded place. This is the consequence of working with a science that has no determined genus. This non-categorical, universal science is literally applicable to *anything*.

5.1 MF Dyads are Timelike

One of the Leibniz's many talents was in developing simple and efficient notation. For example, his notational scheme for the Calculus is still the one in use today. Thus, we stick as close as possible to Leibniz's original notation for his geometry of situations. Using Leibniz's notation is one way of capturing the flavour of his thinking. However, a major necessary modification of Leibniz's geometry is to consider all lines as *oriented* entities. In this context, the line AX becomes a totally different animal **from** XA .

Consider now the mF typed line. In Leibniz notation, this can be denoted as AX . Now the dominant concept to take into account is that A is the (accidental) starting *situs* for the construction. Thus, one can say that for the line AZ is *like* AX because they share the same starting *situs*, i.e.

$$AX \gamma AZ \tag{4}$$

In fact, AX and AZ will be indistinguishable. Moreover, since A is the starting point (starting *situs*) then X and Z must both be *after*² A . In addition, to avoid that X and Z be distinguished from each other, although both after A , neither must be before or after the other. This means that, in general, any line of the form AX will always point in the same general direction relative to the starting *situs* A . We see here the beginning of a generic notion of **time**, where AX denotes a cone of timelike lines. We could even say that if the space had an inner product, the inner product $AX.AZ$ would always be positive definite. Any angle between AX and AZ would be less than a right angle.

5.1.1 The Generic Partner of MF Dyads

We now come to the generic partner of AX . The generic partner will be a line starting from A but ending on the mirror image (X) of X and so will be denoted by $A(X)$. The generic partner $A(X)$ of timelike line AX can be thought of as the *timeline conjugate* of AX .

The fundamental characterising property of timelike mF typed lines can be summed up by the relationship that AX is *like* its generic partner $A(X)$, i.e.,

$$A(X) \gamma AX \tag{5}$$

Introducing *bra-ket* notation, the *ket* becomes $|AX\rangle$ and the *bra* $\langle A(X)|$. Writing the *bra-ket* evaluating to unity and noting that the square of AX is also unity:

$$\langle A(X) | AX \rangle = 1 \text{ and also } (AX)^2 = 1 \tag{6}$$

Recognizing that AX corresponds to the gender coding MF and the equivalent corresponding RNA coding letter **a**:

$$(MF)^2 = a^2 = 1 \tag{7}$$

The MF construct and its adjoint is illustrated in Fig. 4(a). In the case where the dyads AX and its adjoint $A(X)$ are equal, they can be said to equal the imaginary oriented number j , which has the property, that it **squares** to $+1$.

5.2 FM Spacelike Dyads

Unlike timelike lines, spacelike lines start at no determined *situs*. Thus, a spacelike line will be labelled YB , which denotes a dyad with determined *situs* B on the target side. Spacelike lines can share a common

² After (or before) relations do not necessarily violate FC provided that, in the long run, the entity is not after *all* other entities in the system or organism. In addition, not all constituents are necessarily perfectly synchronous with the "now." The various parts of the system can be thought of as closely clustered *around* the systemic "now." The now can even be thought of as some kind of brane. What is not immediately synchronous will have imaginary status, the synchronous being the real part. This is how Now machines operate.

determined *situs*, but only as an end *situs*. Two spacelike lines YB and ZB can be said to be *like* each other written as

$$YB \sim ZB \quad (8)$$

The dyads YB and ZB are like each other because they share the same relationship to the *situs* B . This means sharing the end *situs* B and, as for the MF case, the *situs* Y and Z must have the same *after* relationship with respect to B . This time the after relationship is spacelike rather than timelike.

5.2.1 The Generic Partner of FM Dyads

Unlike timelike lines, the fundamental characterising property of a spacelike line YB is that it is *not* like its generic partner $(Y)B$. To be like the spacelike line YB , the generic partner $(Y)B$ would have to be to the right of B . Instead, generic partner $(Y)B$ is diametrically opposed to YB , having the same orientation but opposite sense. Thus, $(Y)B = -YB$.

The YB (or MF) construct and its adjoint is illustrated in Fig. 4(b). The diagram also illustrates the special case where YB could become equal to the oriented imaginary number \hat{j} . This imaginary entity has the property that it squares to -1 just like the ordinary scalar imaginary number 'i' in complex number analysis. The adjoint of the oriented number \hat{j} is equal to its conjugate which, of course, equals $-\hat{j}$.

Writing the *bra-ket* version as evaluating to unity as usual but this time noting that the square of the spacelike line YB is minus unity

$$\langle (Y)B | YB \rangle = 1 \text{ and also } (YB)^2 = -1 \quad (9)$$

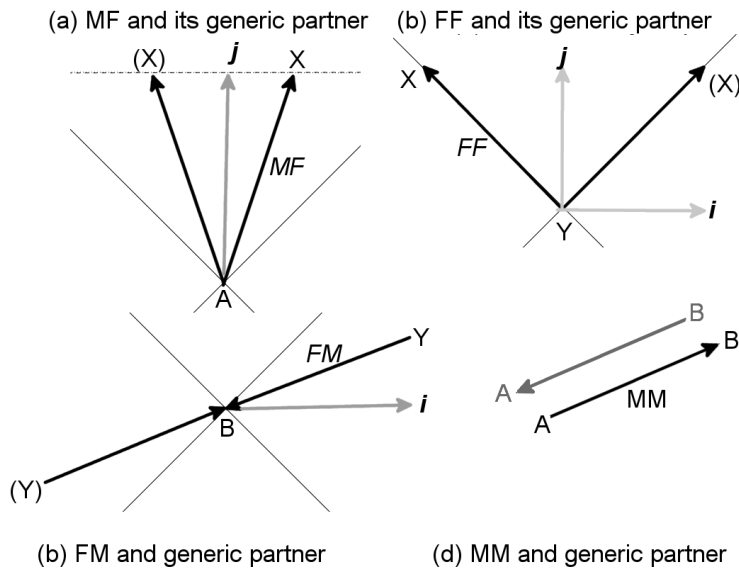


Fig. 4. The MF , FM , FF , MM dyads and generic partners

5.3 FF Lightlike Dyads

Traditional Hilbert space and even GA based on the Clifford algebras are all founded on geometric signatures of the form (p,q) where p is the number of basis vectors that square to $+1$ and q the number that square to -1 , or in other words, the number of \hat{j} and \hat{i} oriented imaginary number basis elements. Sometimes the signature (p,q,r) is mentioned where r is the number basis vectors that square to zero. However, this latter case is invariably quickly dismissed as degenerate. Thus, one can see that traditional geometry is all based on basis elements that have one determined *situs* and one non-determined *situs*; in other words, the basis elements are all binary gendered MF or FM . The other two possibilities of FF and MM gendered basis elements are ignored in traditional geometry. The grand geometric message of our work is to show how Nature uses all four basis types to construct an infinitely richer world of possibilities than the half world of traditional geometry.

For the case in hand, we are considering the FF typed dyad that corresponds to lightlike lines. One can talk about lightlike lines, and the other types of line, from what is known about the traditional approach to spacetime geometry. However, to remain true to Leibniz's original intention, the discussion should be based on first principles and carried out without number. The approach adopted here is a mishmash of the two. Nevertheless, the author does believe that eventually an explanation can be developed that formally explains these concepts

without appealing to such things as the inner product “if it existed,” and so forth. Of course, the presentation would have to be non-axiomatic. These are early days in attempts to formalise material based on the non-dualist paradigm. Meanwhile, we venture on.

A lightlike line in Leibniz notation will be denoted by XY and so will have undetermined start *situs* as well as undetermined end *situs*. If line XY is like WZ i.e.

$$XY \gamma WZ \quad (10)$$

then XY will have no determined relation to WZ as all the *situs* involved are undetermined. This requirement would be violated if the two lines had any correlation with each other. If there were an inner product, this non-interacting condition would be that the inner product $XY.WZ$ be zero.

Consider the thought experiment of choosing a lightlike line XY without actually consummating the choice and repeating the exercise by choosing WZ . The outcomes of the two non-extractive choices could be exactly the same line. This only goes to show that any line is like itself and so the inner product of a XY with itself would be zero.

5.3.1 The Generic Partner of FF Dyads

We now turn to the generic partner $(X)Y$ of the line XY and, in particular, whether the lines are correlated in any way. The two lines now share the same target Y and so, taken together, they are correlated in some way. Since XY and its generic partner $(X)Y$ share the same start *situs*, they will be correlated and so have a non-zero inner product.

In a space already equipped with an MF and an FM dyad basis, the two corresponding oriented imaginary numbers \mathbf{j} and \mathbf{i} can be used to define another oriented imaginary number \mathbf{k} where

$$\mathbf{k} = \mathbf{j} - \mathbf{i} \quad \text{and so} \quad \mathbf{k}^2 = (\mathbf{j} - \mathbf{i})^2 = 0 \quad (11)$$

The generic partner \mathbf{k} of \mathbf{k} can be expressed as $\mathbf{k} = \mathbf{j} + \mathbf{i}$ and so the inner product will be

$$\mathbf{k} \cdot \mathbf{k} = 2 \quad (12)$$

Fig. 4(d) illustrates the oriented imaginary number \mathbf{k} construct. The diagram also shows that, except for a possible normalisation factor, the dyad XY is identical to \mathbf{k} . The same thing applies for the generic partner of $(X)Y$ which will be identical to \mathbf{k} . This leads to a very important observation. Proceeding classically, a vector space $\mathcal{V}(1,1)$ can be constructed using one \mathbf{j} dyad and one \mathbf{i} dyad as basis. This would be a two dimensional Minkowski space. An alternative null basis $N = \{ \mathbf{n}_0, \mathbf{n}_1 \}$ would be simply normalised versions of \mathbf{k} and \mathbf{k} defined by

$$\mathbf{n}_0 = \mathbf{k}/\sqrt{2} \quad \text{and} \quad \mathbf{n}_1 = \mathbf{k}/\sqrt{2} \quad (13)$$

Linear combinations of the two elements of the null basis N spans the same space $\mathcal{V}(1,1)$ as the non-null basis $M = \{ \mathbf{i}, \mathbf{j} \}$ even though traditional mathematics tends to treat the null-basis in an inferior way. For example, the signature of a space is expressed in terms of the non-null basis not the null basis.

The *bra-ket* form for the XY dyad and its generic partner $(X)Y$ is

$$\langle (X)Y | XY \rangle = 1 \quad \text{and also} \quad (XY)^2 = 0 \quad (14)$$

where $XY = \mathbf{n}_0$ and $(X)Y = \mathbf{n}_1$.

5.3.2 Contravariant, Covariant, and Invariant Properties of MF , FM , and FF

To each binary type, there is a generic partner that shares the same source *situs*. The generic partners of MF , FM and FF can be written as $M(F)$, $F(M)$ and $F(F)$ respectively. In passing, note that there is a *contravariant* relationship between the MF dyad and the $M(F)$ generic partner. This can easily be seen in Fig. 4(a). Rotating the MF gendered AX dyad clockwise results in its generic partner $A(X)$ rotating anti-clockwise. Likewise, there is a *covariant* relationship between FM and its generic partner $F(M)$ as seen in Fig. 4(b). Rotating FM in one-direction results in its generic partner rotating in the same direction. In the case of FF and its generic partner $F(F)$, the relationship between the two is fixed, as no relative motion is possible. This will be an *invariant* relationship.

5.4 MM Singular Dyads

We have so far considered three of the four possible dyad types, the MF , FM , and FF , gendered vectors which we claim provide the generic geometry semantics to the \mathbf{a} , \mathbf{g} , and \mathbf{u} , letters of the genetic code. Traditional analytic geometry is founded wholly on MF and FM typed vectors. A fundamental characteristic of these vectors is how they square, MF squaring to +1, FM squaring to -1. The FF typed vectors are then treated as subordinate to the MF and FM typed vectors. FF typed vectors square to zero, but are not orthogonal to MF and

FM vectors and so cannot be used as basis vectors of an orthonormal basis that includes *MF* and *FM* typed basis vectors. However, an *FF* typed vector can be used with its adjoint partner as a 2D null basis for \mathcal{V}^2 .

The fourth type of dyad is the *MM* typed vector, the *singular vector*. Similar to the *FF* dyad, the *MM* dyad also squares to zero and hence is orthogonal to itself. However, the *MM* dyad is also orthogonal to any other vectors present. As a token gesture, traditional analytic geometry admits *MM* as a possible basis vector type in an orthonormal basis set. Thus a vector space $\mathcal{V}(p,q,r)$ will have a metric signature consisting of p basis vectors that square to $+1$, q that square to -1 , and r of the *MM* typed vectors that square to zero. Having thus recognised the possibility of *MM* typed vectors, the usual next step is to declare that any vector space will be considered degenerate if it includes them. Thus, a vector space $\mathcal{V}(p,q,r)$ with a non-zero value of r will, in general, be ignored by mainstream mathematics.

The overall picture of the traditional analytic approach to geometry is only to seriously consider two of the possible four generic vector types. In effect, the only linear metric vector spaces seriously considered by traditional mathematics are of the form $\mathcal{V}(p,q)$. From our universal geometric perspective, this traditional approach to geometry can be judged as rather “half brained.” It is like the stroke patient that only has a fully functional left hemisphere and so only eats food on the right side of the plate. One could say that the patient considers food on the other side of the plate as “degenerate.” In psychology, the phenomenon is called *hemineglect*. Our message is that not only do right side stroke victims suffer from hemineglect, but that the whole of traditional analytic mathematics also exhibits a similar cognitive deficit³. Traditional left side mathematics only knows how to consume *FM* and *MF* typed geometric entities. On the other side of the plate are the *FF* and *MM* typed entities. Analytic mathematics considers the *FF* type as merely of subsidiary **stratus** and the *MM* type as totally degenerate. The task in hand is to show how the degenerate *MM* typed dyad plays a crucial role in the universal synthetic geometry based on all four generic vector types, each being of equal importance as the others.

So far, except for the foundational gender construct, our coverage of the first three generic vector types has not required any significant innovation on existing geometric constructs. However, when it comes to the enigmatic *MM* typed vector, a significant geometric innovation is necessary. In order to avoid the hemineglect problem that afflicts traditional mathematics, we need to cure the cause of the affliction. The cause of the malady is due to traditional mathematics’ obsession with *value* and the refusal to deal in an equitable manner with the other side of the equation. In the real world, every value must have a *placeholder*. Our tenet is that the universal algebra of value-placeholder dialectics is the **generic-cum-genetic** code based on gender. The gender opposition between masculine and feminine is not just a placeholder-value oppositions as both the masculine and the feminine can each take on a value or placeholder role. The gender construct is simple but not simplistic.

The *MF*, *FM* and *FF*, types are all less determined versions of the *MM* type. Using Leibniz notation, a singular *MM* typed dyad will be denoted by AB where the two ends of the dyad both have determined *situs*. Ultimately, this is the fate of any dyad. In any singular determined instance, no matter what its disposition expressed as gender, any dyad will collapse to appear as an AB dyad.

The universal nature of AB can be demonstrated by its likeness to any other similarly determined dyad CD . We can write

$$CD \gamma AB \tag{15}$$

5.4.1 The Generic Partner of *MM*

Since there is no way to distinguish the CD dyad from AB , the likeness relation γ can be taken as an identity (but not in the usual abstract set theoretic relational sense). There is one exception. The only dyad with determined *situs* at both ends that is objectively distinguishable from AB is the dyad BA . However, the distinguishability of AB from BA is only operational when both dyads are simultaneously present. They cannot be distinguished by any difference in attribute. Here, the Leibniz scholar would recognize that Leibniz frequently makes this demand when reasoning about two entities; both must be simultaneously present.

The BA dyad will be the generic partner of the AB dyad and is distinguished by that fact that AB and BA do not satisfy the likeness relation γ . For the other three dyad types, the likeness relation could be characterised in terms of the inner product. However, for the singular dyad the inner product is always zero between all dyads present including the singular dyad itself. In other words, the *MM* gendered singular dyad is effectively devoid

³ If the problem domain is restricted to the world of traditional stand-alone mechanical structures then analytic geometry may be perfectly suitable for the task. Analytic mathematics will not display any “cognitive deficit” in these cases as the object of study itself is assumed to be totally cognitively degenerate. The limitations of analytic mathematics only conceiving a half world only become apparent in the study of self-organizing organisms. Such systems cannot afford to be cognitively degenerate and so an all-embracing relativistic “right-side” perspective on reality becomes necessary.

of quantifiable value. However, one must admit that BA is a dyad of opposite sense to AB, which implies that there is a geometric difference between the two.

Using *bra-ket* notation, AB makes up the *ket* $|AB\rangle$ and BA the *bra* $\langle BA|$. As to why this is not the other way around where BA is the *ket* and AB the *bra*, the only objective way of settling that question is to accept the combination chosen as according to a *convention*. If Nature were based on such a convention, this would imply that the inanimate we live in is based on a preferred orientation. In actual fact, we do know that Nature abounds with orientation and handedness conventions. The Arrow of **Time** in physics is an example of Nature's most profound orientation conventions. Such conventions would seem to fly in the face of the FC principle, which abhors absolutes of any kind. We will not enter into this perplexing matter here. We simply comment that in order to achieve FC such relativistic conventions seem indispensable for the cause.

Continuing with the *bra-ket* notation we note that since AB and BA are singular dyads their inner product must be zero, i.e., the *bra-ket* satisfies:

$$\langle BA|AB\rangle = 0 \quad (16)$$

Our use of the *bra-ket* notation is to provide a mechanism showing the *bra* corresponding to the body and the *ket* to the monad side of an organic unity. However, this time the evaluation of the *bra-ket* does not lead to a unit but a zero. Rather than a formula demanding an organic unity, the formula demands an organic difference. AB and BA do not harmoniously interact; they are so different that they do not interact at all. This reveals another side to the body-monad relationship. Intuitively one can say that, in this context, body corresponds to *Ground* and monad corresponds to *Figure*. What is Figure, what is Ground is a subjective matter, and Figure-Ground reversal may be possible. The objective constraint is that Figure and Ground do not mix.

Here we see a glimmer of hope for the much-neglected singular dyad. The singular dyad holds the potential to introduce a new geometric construct that is totally lacking in analytic geometry, the Figure-Ground construct.

5.4.2 Where is the *MM* dyad located?

It goes without saying that really to understand something in geometry one must understand it geometrically. **This** probably applies to understanding anything in mathematics. It is only geometric understanding that fully expresses the semantics of meaning, not abstract symbolism. Thus, we have the *MM* dyad as a line with determined *situs* at both ends. The whole *MM* line then must have determined *situs*. The term *situs* is used here as implying some kind of generic location. What now follows is the question: "What is the generic location of an *MM* typed dyad?"

Unlike the other three dyad types, the *MM* gendered dyads cannot be interpreted as a determined value. Nevertheless, if the singular dyad construct is not to be degenerate then it must be capable of being interpreted as *something*. The only viable and tractable alternative is to interpret this type of dyad as a *placeholder* for value. Whilst analytic mathematics is all value oriented, synthetic mathematics corrects the one-sidedness. Synthetic mathematics must not only embrace value but *situs* of value. The apparently "degenerate" singular dyad starts to reveal its fundamental organisational **role** in synthetic geometry. There are two roles, one implicit and the other explicit.

Consider a gender typed dyad *D*. The explicit typing of *D* can be any one of the four gender typing possibilities including the explicit typing *MM*. Thus, *D* can represent any one of four gender types *as value*. The value of the dyad *D* must have a *situs*. The implicit *situs* will be implicitly typed *MM*. The role of the implicit *MM* typing of the *situs* is to provide a mechanism that is potentially capable of indicating whether the dyad *D* belongs to Figure or to Ground. Whether the *situs* belongs to Figure or not will be indicated by the orientation of its implicit *MM* typed. If the *situs* indicates that its value content belongs to Figure, we will say that it has positive *situs charge*. If not, the *situs* will have negative *situs charge* and its content will belong to Ground.

The preoccupation of any organism is to highlight itself as the Figure on stage, not the stage that merely plays the support role for the performance. Thus, positive *situs charge* will be the preferred default. The four possible values occupying the *situs* will be the four determined gendered type *MF*, *FM*, *FF*, and *MM*. The first three of these types will correspond to determined values, In the case of *MM* as a value for the *situs*, the value is much more allusive as this type corresponds to a virtual negation of determined value. The explicit *MM* type signals a complete lack of determined specificity in this regard. One interpretation would be to say that the underlying implicitly typed *situs* is empty of determined content.

Value is located in its *situs* placeholder, but the main question still remains. What are the possible generic locations for the *situs* itself? What is the *situs* of the *situs*? The answer to this question is surprisingly simple. From our previous discussion, there are only four possible generic locations for *any* gender typed dyad, no matter what the binary gender. These locations make up a Three-Plus-One structure where the triadic component is an RGB triad. There are thus three generic locations, the R, G, and **B** locations. These are the phenomenal "imaginary" parts of a generic codon structure. The "real" part is the "white" dyad part. The *MF*, *FM*, and *FF*, typed dyads bring along additional specificity to the colour-coded location. The *MM* dyad behaves in much the same manner except that it is devoid of such specificity. The only specificity enjoyed by the *MM*

dyad is its colour-coded *situs*. There are three such generic *situs*, the Red, the Green, the Blue, and the white generic locations on a generic Three-Plus-One structured codon.

This colour-coded *situs* property of *MM* dyads applies to both animates and inanimates. In the case of inanimates, we propose this as a geometric explanation of the so-called “colour charge” of elementary particles (but at a deeper, more fundamental level). Thus *situs* charge is not only signed it can take on one of three phenomenal values R, G, B, depending on the dyad location in the codon. Henceforth, we borrow the poetic language of Particle Physics and refer to *situs* charge as *colour charge*. We note that any triad can have any of the RGB signed combinations leading to eight possible colour charges for an elementary triad, just like the quarks in Particle Physics. Of course, in our case any quark type structure will not be devoid of internal structure but will consist of three gendered dyads and so make up a “quarklet.”

5.4.3 *MM* dyads for Animates and Inanimates

In the early stages of trying to understand the geometric semantics of the *MM* monad, the author looked at how triads of *MM* monads, as genetic code codons, appeared in Nature. According to our approach, in RNA coding the *MM-MM-MM* triad becomes the *CCC* codon. In animates, *CCC* transcribes to the amino acid proline. Proline has a number of unique characteristics. Formally, it is not even an amino acid, but an *imino* acid. Proline is the only one of the genetically coded molecules that has a cyclic side chain. Due to its unique structure, proline occurs in proteins frequently in turns or bends, which are often on the surface. It has exceptional conformational rigidity compared to other amino acids. All of this makes interesting reading and makes one impatient for the day when Generic Geometry is developed enough to predict all of this structure from generic principles. However, for the moment, reading about the empirically known geometric details of proline is like trying to read tealeaves in a teacup. Proline does seem a bit special though.

In the case of inanimates, generic code triads should lead to matches with the elementary particles of Particle Physics and even predict new ones. There is no transcription stage with inanimates and so the *CCC* triad should directly correspond to an elementary particle. As developed further in Part 2, the *CCC* particle would have zero mass, zero charge, and a spin of unity, thus identifying it with the gluon boson. The gluon is an exchange particle for the strong force and so, once again, even this instance of the generic *CCC* structure is fundamentally associated with systemic rigidity. The gluon is said to be the carrier of “colour charge.” If one managed to get a fundamental generic geometrical explanation for the *CCC* construct, one could start to demystify the empirically derived notion of “colour charge.” Our analysis of the *MM* dyad is offered as a first tentative step in that direction.

6 Generic Geometry in Nature

Analytic geometry is founded on the notion of a vector space $\mathcal{V}(p,q)$ where a vector is defined as a linear combination of $n = p + q$ orthonormal basis vectors and (p,q) specifying the metric signature. The vector space serves as a container for geometric bodies usually defined as point sets. A fundamental feature of analytic geometry is its symmetry and total lack of handedness. Even in the spacetime interpretation of Minkowski space $\mathcal{V}(1,3)$ where there might be an apparent “Arrow of Time” geometric feature along the temporal dimension, the temporal construct has no real geometric meaning, as there is no objective reason why the passage of time should flow in one direction and not the other. As is well known, reversing the direction of time in the analytic mathematics of physics makes no difference to the physics. In brief, analytic geometric spaces have no shape. Only the contrived things that fit into analytic space can have shape. Such contrived shapes have no fundamental geometric significance.

By contrast, the fundamental feature of the synthetic geometry developed here is its systemic asymmetry as indicated in the four elementary geometric forms with gender typing *MF*, *FM*, *FF*, and *MM*. The four elementary forms are illustrated with Leibniz notation in Fig. 5(a). The gender typing can be conveniently illustrated with what we call Heraclitus diagrams as shown in Fig. 5(b). As shown in our previous work, these same diagrams can be used to illustrate the logic of four of the five Stoic indemonstrables as well as the distributed/undistributed typing of the four terms in Aristotle’s syllogistic logic. All of this goes to show that these four generic forms also have fundamental logical interpretations as well as geometric.

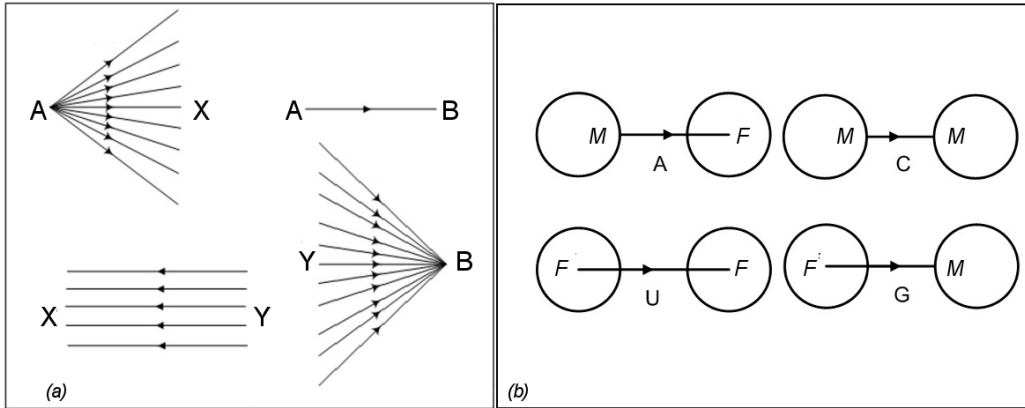


Fig. 5(a) The four elementary forms in Leibniz notation. (b) Heraclitus diagrams for timelike *a* dyads, spacelike *g* dyads, lightlike *u* dyads, and singular *c* dyads illustrating the gender typing corresponding to the Leibniz notation.

The four elementary generic forms are the basic building blocks of any organism, be it animate or inanimate. The asymmetry is so extreme that it may appear as quite odd on first encounter. Instead of a whole space of extent defined as $\mathcal{V}(p,q)$, synthetic geometric of generic forms are (is?) made up as “half spaces.” For example, Fig. 6 illustrates the analytic 2D Minkowski space $\mathcal{V}(1,1)$ interpreted from a synthetic perspective as a combination of cones of timelike and spacelike lines, together with lightlike bundles. Synthetic geometry splits the structure into two, with the elementary monad constituents of this composite monad are (del) on the right and their generic partners on the left forming the corresponding co-monad. The monad and co-monad are both incorporeal. The monad has a corporeal body, which provides the placeholder for the monad acting as value. The same applies to the co-monad. The overall configuration will be as illustrated in Fig. 3. This structure, half idealist, half-materialist, implements a strategy to satisfy the FC demands of non-dualism. Satisfying FC is a generic problem and calls for a generic solution. The author claims that bilateralism of the hemispheres and body sides, is an example of the emergence of this generic solution. The generic form considered here can be expressed as $\mathcal{G}(aug)$, where the codon *aug* corresponds to the start codon in the genetic code. When not in the start position, the *aug* codon also uniquely codes the amino acid methionine.

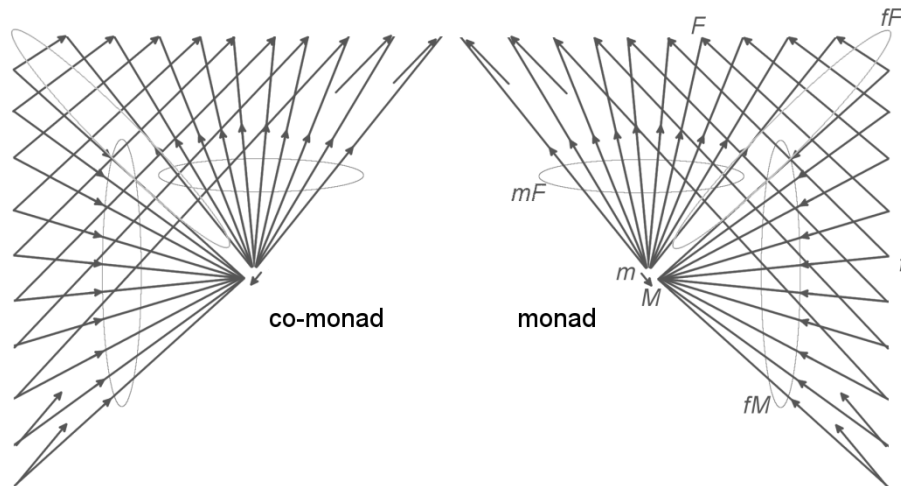


Fig. 6. The synthetic geometry version of spacetime geometry consists of a monad constructed from a single timelike cone of *mF* dyads, a single spacelike cone of *fM* dyads, and a lightlike bundle *fF* dyads all making up an AUG codon. The co-codon in this case is the mirror image

6.1 The geometry of the *aug* codon

In Fig. 6, the *fF* lines types can be meshed with the *mF*, and *fM* to form a spacetime diagram. A more detailed analysis would show that the *mF* and *fM* type lines would be seen as having determined length but only dispositional angle, whilst the *fF* lines had indeterminate length but determined angle. The various degrees of freedom cancel themselves out to produce the $\mathcal{G}(aug)$ form. The three dyads in the form correspond to the triad part of a Three-Plus-One form, the “imaginary” part. The three dyads can be thought of as the “observable” or “phenomenal” aspect of the geometric entity being determined. Together they determine the specificity of the “real” part of the Three-Plus-One form,

6.2 Elementary Generic Forms

Each elementary geometric form \mathcal{G} in Generic Geometry is characterised by a signature made up of three letters from a four-letter generic alphabet $\mathbf{A} = \{\mathbf{a}, \mathbf{u}, \mathbf{g}, \mathbf{c}\}$. These generic forms can be referred to as *generic codons*. A generic codon with signature *ggg* can be written as $\mathcal{G}(ggg)$. The closest analytic Cartesian version of this triadic form is the 3D anti-Euclidean space $\mathcal{R}(0,3)$. The generic version of the 3D Euclidean space version $\mathcal{R}(3,0)$ would be $\mathcal{G}(aaa)$. However, such comparisons can be misleading as the semantic interpretation of generic geometric forms is quite different from their scalar analogues in the Cartesian worldview. In the spacetime geometric interpretation of the generic alphabet \mathcal{A} , the letters \mathbf{a} , \mathbf{u} , \mathbf{g} and \mathbf{c} can be interpreted as denoting timelike, lightlike, spacelike, and singular vectors.

Also of interest is the generic form $\mathcal{G}(aug)$. The Cartesian version of this form is the Minkowski space $\mathcal{R}(1,1)$ which is the 2D expression of spacetime geometry. In the case of animates, the codon *aug* codes the *start codon*. Unlike Cartesian geometry, the ordering of the basis elements in the generic signature is geometrically significant. For example, by changing the order of the letters in the start codon *aug* one could get *uag* or *uga*, which for animates code *stop codons*. The geometric semantics of stop codons has not yet been addressed in our work so far.

In animates, the generic code is known to be redundant. The only non-redundant coding is the *aug* start codon, which also codes the amino acid methionine, and the *ugg* codon that codes the amino acid tryptophan. Interestingly, the nearest Cartesian version of the generic form $\mathcal{G}(ugg)$ would be the de Sitter space defined as a 2D manifold in $\mathcal{R}(1,2)$. De Sitter space can be interpreted as the vacuum solution to the equations of General Relativity, which could indicate that the General Theory might have universal, generic interpretations that go well beyond that of physics. In other words, General Relativity not only applies to the inanimate that we live in but to the organisational semantics of all animates as well.

To explain the redundancy of the genetic code the author proposes the conjecture:

The FC Degeneracy Conjecture states that the observable degeneracy of the genetic code does not result from any quirky biochemistry but is due to the underlying generic geometric structure based on FC that it implements. Codons are grouped together (hence “degenerate”) to form geometric wholes satisfying FC. In the cases of AUG and UGG, only one codon is necessary for a FC compatible geometry. In some case, up to six codons are necessary to form an FC compatible geometry.

Examples of where a geometric form would violate FC would be the lack of a point at infinity construct and the lack of a truly generic origin. It can be argued that the geometric forms $\mathcal{G}(aug)$ and $\mathcal{G}(ugg)$ both satisfy this requirement. However, Euclidean and of course **Anti-Euclidean** space violates FC on both these counts. The generic version of Euclidean space is $\mathcal{G}(aaa)$ which, for animates, codes the amino acid lysine. The coding is doubly degenerate as the generic form $\mathcal{G}(aag)$ also codes lysine. The generic version of Anti-Euclidean space is $\mathcal{G}(ggg)$ which for animates codes the amino acid glycine. Four different codons code glycine as illustrated in the Heraclitus diagram of Fig. 7(b).

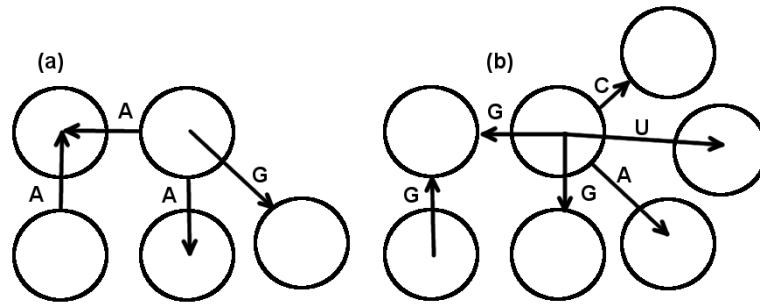


Fig. 7(a) The amino acid lysine is coded by *aaa* and AAG (b) glycine is coded by *ggg*, GGA, GGU, and GGC

Conventional analytic geometry is not very discerning when it comes to representing Euclidean space as $\mathcal{V}(3,0)$ or as $\mathcal{V}(0,3)$. The space with the positive metric (is) $\mathcal{V}(3,0)$ would seem to be the most suitable but is made up of three lightlike lines, which does not make much sense. The version $\mathcal{V}(0,3)$ is made up of three spacelike lines and so would appear to be the most physically plausible but has negative metrics thus meriting the space with the name Anti-Euclidean space. Mathematicians note that $\mathcal{V}(3,0)$ and $\mathcal{V}(0,3)$ are isomorphic and so the two spaces are considered mathematically equivalent.

Analytic geometry provides a simple container framework for inanimate non-organised substances. Generic geometry addresses a totally different reality, the reality of the self-organising, self-maintaining entity suitably qualified by the dispositional geometry of Organisms. In the generic context of Organisms, analogous generic versions of analytic geometric forms such as $\mathcal{V}(3,0)$ and $\mathcal{V}(0,3)$ take on entirely new meanings. Thus, in the case of animates, the generic version of $\mathcal{V}(0,3)$ becomes the $\mathcal{G}(ggg)$ coding of the amino acid glycine. If our FC Degeneracy Conjecture were correct, this primordial fragment of 3D Euclidean geometry would violate FC. For example, $\mathcal{G}(ggg)$ would lack a generic origin as well as the point at infinity construct. The Degeneracy Conjecture postulate for animates is that the geometry of $\mathcal{G}(ggg)$ must be augmented to a larger projective space necessitating degeneracy of the code, as illustrated in Fig. 7(b). This is not the place to attempt any detailed mathematics, but projective methods would suggest that **point in could be** modelled by dyadic line segments in the total projective space where the lines segments have zero metric, just as in conventional conformal geometry. Points become special cases of line segments where the metric of the line segment becomes zero. The necessary mathematics would be much like conformal geometry together with the added nuance of **C** type dyads. The details of how to properly handle **C** type dyad is left as an open question.

Thus, for animates, a Euclidean form like $\mathcal{G}(ggg)$ becomes merely a dispositional constituent brick like many **others that** make up the whole organism. According to Leibniz's vision, this geometric form corresponds to an incorporeal construct that he called a monad. Associated with each monad was a corporeal entity, the body. Note that the generic form $\mathcal{G}(aaa)$ for animates has a quite different extension space **from** that of $\mathcal{G}(ggg)$. Generic geometry has a much finer structure than analytic geometry.

For animates, the degeneracy of the code in three cases allows six different codons to code a single amino acid. Leucine is such an example resulting in the amalgamation of six different Heraclitus diagrams as in Fig. 8. Since the composite diagram must be built from unambiguous triads, the composite Heraclitus diagram in this case divides into (**has**) two different structures. The structure seems to impose some sort of bimodal geometry. The details of this new **kind** of geometric form should prove very interesting.

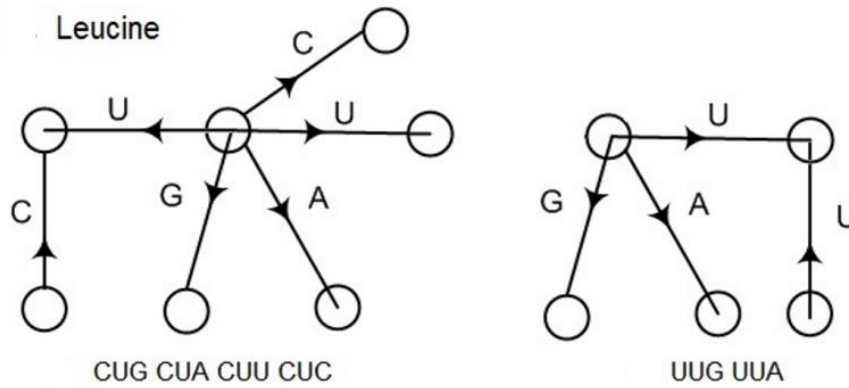


Fig. 8. The composite Heraclitus diagram for the amino acid leucine indicates an intricate bimodal geometry. All six codon possibilities code leucine.

In the case of *inanimates*, degeneracy of the genetic code is not possible as the coding and the coded are the (the) same. Inanimates must exploit a different strategy in order to achieve FC. In principle, the generic form (ggg) would correspond to an elementary particle of some kind that may or may not exist.

6.3 Why are all coded amino acids left-handed?

The matter and antimatter asymmetry in inanimates is matched by the dramatic asymmetries observable in animates. This observation has its roots in an incredible discovery first reported by Pasteur. Working with organic substances, certain chemical processes carried out in the laboratory produce L-amino acids. The result is always a 50-50 mix of left and right-handed amino acids. Pasteur observed that this was not the case when living organisms produce the amino acids. Biosynthesis produces amino acids for the synthesis of protein. All of these amino acids, without exception, are left handed. All proteins produced from such amino acids are also left handed. Similarly, the D-amino acids that are involved in the biosynthesis of D-sugars all turn out to be right handed. In short, the organisation of biological life seems to be dominated through and through by handedness. Evolution critics exploit this fact to attack the primeval soup hypothesis for the spontaneous creation of life. The critics argue that it is not enough to start with, a soup of amino acids to build the proteins necessary to kick start life, as all the amino acids have to be left handed. In fact, the presence of just one single amino acid molecule of the wrong handedness in a sequence is enough to upset the applecart irretrievably. Such a tainted protein would be dysfunctional.

What is the explanation of the homochirality of amino acids and why are (the) only the L-amino acids coded and the D-amino acids not coded? Some writers (Mason, 1991) (McManus, 2002) suggest that this mysterious homochirality of amino acids, proteins, and sugars in living organic material can be explained by physical asymmetries at the level of sub-atomic particles. From this perspective, the asymmetries in Nature would sweep right across the sub-atomic world, up the scale through chemistry to biochemistry, and onto the macro world of Nature's complex biological organisms continuing right up to Homo Sapiens. Handedness would be everywhere and at all scales. A noble universal principle would seem to be at play here; the problem is to understand it.

There have been many attempts to explain the asymmetries in Nature's handedness enigma. A constant refrain by many writers, McManus included, is that biological asymmetries, even lateralisation of the brain, must somehow provide a favourable functional advantage in Evolution. Evolution results in the functional advantage being coded into specific genes for the various asymmetries.

A central theme of our project is to look to the things that are impervious to Evolution and without which Evolution would not be possible. We refer here the *generic-cum-genetic* code that does not evolve and must be present from the beginnings. Organisms exploiting and implementing this code are capable of maintaining material existence without violating FC. In all cases, the excursion into material existence would be of finite duration with each determined beginning necessarily leading to a determined end. Nevertheless, the excursions are real. All animates and inanimates exploit this generic calculus in order to *be*. In this perspective, the chirality of Nature did not evolve. Instead, chirality is an integral part of a universal generic calculus, a calculus that expresses itself as a universal geometric algebra.

We argue that chirality in Nature did not evolve. It emerged (exists?) as an essential part of the generic geometry necessary for achieving FC. The fundamental organisational strategy for achieving and ensuring FC is that everything should be left "half-baked" so to speak. Only the bare minimum needs to be declared explicitly. Only one-half of the jigsaw need ever be explicit. In a reality dominated by the FC requirement, the other half

will inevitably fall into place. In Fig. 2, only the right side monad corresponding to the ket $|\Omega\rangle$ need be explicit. On its own, the ket violates FC and so needs to be complemented with its generic partner corresponding to the bra $\langle\Sigma|$ satisfying the bra-ket constraint in (17). The bra-ket entity is left-handed and so violates FC by declaring a distinguished chirality. It goes without saying that there must also be the entities with the opposite chirality. The bra-ket entity $\langle\Sigma|\Omega\rangle$ demands the existence of a full chiral partner as illustrated by the shaded entity.

It is this kind of argument that we propose for explaining not only the chirality in Nature but also more concretely why the coded L-amino acids are all left handed and the uncoded R-amino acids are not. The coded L-amino acids are playing the systemic organisational role of monads. Everything else falls into place as there is nowhere else to go, at least not without violating FC. That provides an explanation of the ontological role of chirality and why D-amino acids are not explicitly genetically coded and are all right handed. (Doug, Fig. 2 mentioned above is the RGB triad diagram, page 9 – is this what you are referring to? Also “the bra-ket constraint in (17)” – where is (17)??)

7 Conclusion

Leibniz envisaged a greatly simplifying metaphysical science that would ultimately be expressed in the form of a geometric algebra that could explain the generic forms of Nature in a few letters. Leibniz saw this as applying equally to the inanimate as to the animate. In this paper, we have explained how such an approach can be bootstrapped from the requirement of FC: The requirement is that a system must not tolerate a situation where any systemic entity is in a privileged position relative to all other entities of the system. From this principle alone, the whole metaphysics can be bootstrapped. The resulting science, algebra, and geometry are of a different kind from the traditional sciences. The science is operational and resembles in some ways that of the Operational Calculus of Heaviside but on a much grander stage. The end result is a geometric algebra approach based on a four-letter alphabet. The paper claims an exact match between the triads of the resultant four-letter algebra and the codons of the genetic code and thus goes a long way to substantiating Leibniz’s original vision statement.

A novel feature of Leibniz’s metaphysics is his doctrine of the harmony between his incorporeal monads and corporeal bodies. This paper provides the first tractable explanation of how this may work based on different kinds of adjoint relations as a way of implementing non-duality. The approach naturally leads to systemic bilateralisation. Systemic bilateralisation is a fundamental organisational form observable in emerging biological organisms, if not for even the inanimate realm.

In this paper, we have expanded Leibniz’s tentative notational scheme for developing the necessary geometry without number. There is very little in the way of clever innovative contributions in this paper. Our approach is not inventive or ‘fictionalist’ like many theories of traditional science and even mathematics. Our previous work has followed and discerned the fundamental alignments with Aristotle’s syllogistic logic and the Stoic approach to philosophy and logic. The only major innovation has been on founding the whole system on the principle of ontological gender. But even the gender construct itself is not new, and can be found in most ancient cultures. Our only contribution to the gender construct is to present it in a more formal way as a fundamental expression of the opposition between *to have* and *to be*.

The fundamental contribution in our work is to show how all of these apparently disparate pieces of the puzzle fit together. The work reported here provides the first tractable response to Leibniz’s very demanding vision. Not only must Leibniz’s proposed universal science provide the generic geometric semantics of the animate world, it must also be applicable to the world of inorganic material substances. In other words, the genetic code must be a universal generic code, applicable to the physics of elementary substances. Of fundamental importance is that the resulting Leibniz style science is tractable and leads to precise scientific predications and explanations. For example, in this paper we readily explain the mysterious homochirality of biologically produced amino acids.

This paper concentrates on the generic structure of entities that have at least one level of transcription between the letters of the code and the functional substance, and those entities where there is no such transcription. These are called animates and inanimates respectively. Biological organisms are examples of animates, our universe is an example of an inanimate. Later work will provide a more detailed treatment of inanimates. This will lead to a Generic Model of the elementary particles of physics as the generic, non-empirical alternative to the Standard Model.

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